WOOD RESEARCH
51 (4): 2006
1-10

AN ATTEMPT AT THE VERIFICATION OF THE MODEL OF WOOD HYGROMECHANICAL BEHAVIOUR

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ABSTRACT

On the basis of a four-parameter rheological model taking into consideration strains resulting from moisture content alterations, the authors calculated strain values of the creeping wood in non-stationary moisture content conditions which were compared with the strains determined experimentally. An attempt was made to describe, with the assistance of the model of wood hygromechanical behaviour, the determined differences between the results of a purely rheological theory and experimental results. Values of the coefficients of hygromechanical coupling were calculated when beech wood was subjected to radial tension in conditions of wood changing moisture content within the range of hygroscopicity.

KEY WORDS: hygromechanical strain, mathematical model, rheological model, beech wood

INTRODUCTION

Nearly in all areas of scientific research it is possible to notice a certain characteristic pattern of successively alternating cycles when scientists appear to turn from theory to experiments and vice versa. As a rule, results of experiments constitute the base for creative theoretical research. On the other hand, theoretical considerations often help design new, cognitively important experiments whose results either invalidate or confirm the existing theories and, in so doing, provide fresh impulses for further development of these theories.

Theories are also very useful for purposes of design work or teaching where they allow diagrammatic or model presentation of the essence of diverse phenomena.

There are many theories describing the behaviour of wood subjected to tension which is simultaneously changing its moisture content (e.g. Bažant 1985, Grossman 1976, Hunt 1996, Joyet et al. 1992, Ranta-Maunus 1975, 1994, Rybarczyk 1973, Rybarczyk and Ganowicz 1974, Salin 1992, Yahiaoui 1991). The simplest of the above-mentioned theories appears to be the one developed by Rybarczyk (1973) and later interpreted more comprehensively by Ganowicz (1974). The hygromechanical model of properties of wood materials presented in these studies is based on the separation of the total material strain (ε) into three constituents of different origins: mechanical strain (ε_M) - derived only from the acting stresses (with the possible inclusion of the time of their action), moisture strain (ε_W) - resulting only from changes in moisture content and hygromechanical strains (ε_p) - developing, as if it were, additionally as a result of the simultaneous action of stresses and changes in moisture content, hence:

$$\varepsilon = \varepsilon_M + \varepsilon_W + \varepsilon_p. \tag{1}$$

According to this model, this constituent, referred to as hygromechanical strain, generally speaking, is described by the relationship:

$$\varepsilon_P(t) = \int_{0+}^{t} \sigma(\tau) * \frac{dF\big(v(t)-v(\tau)\big)}{d\big(v(t)-v(\tau)\big)} * \frac{dv(\tau)}{d\tau} * d\tau, \tag{2}$$

in which:

t - time,

τ - a variable having the character of time,

σ (τ) - stress,

F(v) – function of hygromechanical strains characterising material properties,

v(t) – hygromechanical factor which is the sum of the absolute values of moisture content changes (Δw_i):

$$v = \sum|\Delta w_i|, \tag{3}$$

for the continuous function describing changes in the material moisture content (w(t)), defined by the formula:

$$v(t) = \int_{0}^{t} \left| \frac{dw(\tau)}{d\tau} \right| d\tau \tag{4}$$

For the linear function of hygromechanical strains:

$$F(v) = av, \tag{5}$$

in which ‘a’ is, the so called, coefficient of hygromechanical coupling, formula (2) is reduced to following form:

$$\varepsilon_P(t) = a\int_{0}^{t} \sigma(\tau)\frac{dv(\tau)}{d\tau}d\tau = a\int_{0}^{v} \sigma(v')dv' \tag{6}$$

In the case of a stress constant in time and moisture content either growing or falling by Δw, the hygromechanical strain determined by a simple formula (7) is proportional to stresses and the absolute value of changes of the moisture content and the coefficient of hygromechanical coupling:

$$\varepsilon_P = a\sigma|\Delta w| \tag{7}$$

The above-presented theory was – directly after its development – verified on the basis of experimental results available at the time. Those experiments were considered as introductory and their main purpose was to determine the nature of phenomena accompanying moisture content changes of wood stressed across fibres. The following decades provided ample results of experiments as well as simulations of wood viscoelastic behaviour (e.g. Hunt 1997, Mukudai and Yata 1986, Svensson 1996, Toratti and Svensson 2000, Zhuoping 2005). Generally speaking, the most comprehensive numerical data concerning direct experimental results are available in doctoral dissertations (e.g. Moliński 1984, Muszyński 1997, Plenzler 1985). Having at the disposal very precise measurements carried out by Moliński (1984), the authors decided to subject to another verification the model elaborated by Rybarczyk (1973).

The objective of the verification was:

- To compare values of the creeping wood strains in non-stationary moisture content conditions calculated on the basis of a four-parameter rheological model taking into consideration strains resulting from moisture content changes with strains determined experimentally, but also
- To make an attempt to describe, with the assistance of the model of hygromechanical wood properties, the observed differences between the results of a purely rheological theory and experiemntal results.

MATERIAL AND METHODS

Course of the experiments

Moliński (1984) investigated, among others, linear strains of beech wood samples subjected to tension in radial direction with stresses equalling 20, 30, 40 or 50% of the crippling stress at 25% moisture content. The above-mentioned stresses were applied for 48 h and then they were removed and samples were observed for another 24 h. All samples were divided into three series. Samples from the A series were characterised by constant moisture content (1, 10, 18 or 25%), whereas samples from series B changed their moisture content during the period of 24 h from 1% to 10, 18 or 25% and during the following 24 h their moisture content dropped to 1%. Samples from the C series changed their moisture content from 10, 18 or 25% to 1% during the first 24 h and then during the following 24 h their moisture content was brought up again to their initial level. The moisture content was changed by blowing air characterised by appropriately selected parameters. A group of unloaded control samples changed their moisture content according to the appropriate experimental series. The course of the experiment is presented in Fig. 1.

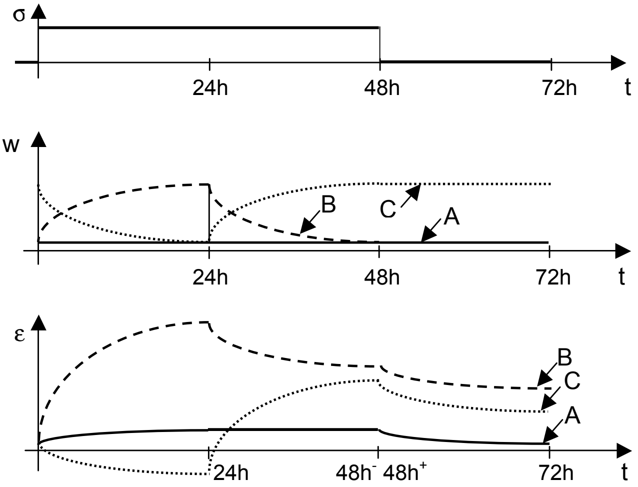

Fig. 1: Schematic diagram of the Moliński' experiment

Theoretical description of the course of experiments

For constant moisture content

In conditions of constant moisture content and temperature, wood behaves like a linearly viscoelastic material, if the acting stresses do not exceed 0.5 of the strength (Grossman and Kingston 1963, Kingston and Budgen 1972). For the above-mentioned conditions, wood strains can be described on the basis of rheological models, e.g. those of Maxwell, Kelvin-Voigt, standard three-parameter model or Bűrgers' model (e.g. Derski and Zięba 1968). The mechanical strains of the examined samples in this experiment were described by the 4-parameter rheological model developed by Bűrgers (e.g. Derski and Zięba 1968, Jakowluk 1993, Fig. 2).The analysis of experimental courses of strains of loaded samples of different but constant moisture content allowed determining values of these four parameters (Tab. 1).

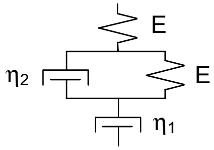

Fig. 2: The rheological model adopted for calculations

Tab. 1: Results of determinations of parameters the Bűrgers' model for beech wood samples subjected to tension in radial direction

Parameter	Unit	Parameter value of Bűrgers' model for sample moisture content of:			
		1%	10%	18%	25%
$1/E_1$	1/MPa	$5.12\cdot10^{-4}$	$6.35\cdot10^{-4}$	$8.06\cdot10^{-4}$	$10.20\cdot10^{-4}$
$1/E_2$	1/MPa	$0.82\cdot10^{-4}$	$1.96\cdot10^{-4}$	$2.78\cdot10^{-4}$	$4.55\cdot10^{-4}$
$1/\eta_1$	1/(MPa·h)	$0.41\cdot10^{-5} = l_1$	$0.71\cdot10^{-5}=l_{10}$	$0.96\cdot10^{-5} =l_{18}$	$1.54\cdot10^{-5} =l_{25}$
$1/\eta_2$	1/(MPa·h)	$0.17\cdot10^{-3}$	$0.58\cdot10^{-3}$	$1.07\cdot10^{-3}$	$1.79\cdot10^{-3}$

For changing moisture content

In order to describe sample strains:

- the authors adopted the description of moisture content strains employing the following function (Moliński 1984, Rybarczyk 1973):

$$\varepsilon_W = k\cdot\Delta w,\ (k=0{,}2), \tag{8}$$

in which
k – is the coefficient of moisture content strains, while
Δw – is the moisture content change,

- the following function was adopted to describe samples stresses:

$$\sigma(t)=\left[1-H\cdot(t-48h)\right]\sigma \quad \text{for } t\rangle 0, \tag{9}$$

where: H(t) is the Heaviside's function (Derski and Zięba 1968),

- the following forms of functions were adopted to describe moisture content changes:
- for moistening:

$$w_+(t)=w_o+(w_k-w_o)\left(1-e^{-r_+t}\right),$$

- for drying: (10)

$$w_-(t)=(w_o-w_k)\cdot e^{-r_-t}+w_k,$$

where:

r_+ and r_- are appropriate coefficients (r_+=0,40 1/h, r_-=0,63 1/h) determined on the basis of runs of real moistening and drying processes in the course of the experiment carried out by Moliński (1984), w_o – the initial moisture content, w_k - the final moisture content,

- the authors further presumed that the parameters in the Bűrgers' model assume – in the case of intermediate levels of moisture content – values resulting from a linear interpolation.

An initial hypothesis was adopted in the performed calculations concerning the possibility of describing the strains of the examined samples by functions (8) and (9) as well as functions resulting from the Bűrgers' model which changes its parameters in accordance with the last assumption. However, it was obvious that the adopted hypothesis was not quite true.

The analysis of the Bűrgers' model, at the changing parameters in relation to the moisture content described by functions (10), allowed describing the total theoretical, albeit purely mechanical, sample strain. For example, in the case of the process of moisture content changes from 1% to 25% and back to 1%, the following formulas were obtained:

$$\varepsilon_T(t=0)=\frac{\sigma}{E_1(w=1\%)},$$

$$\varepsilon_T(t=24h)\cong k\cdot\Delta w(=24\%)+\frac{\sigma}{E_1(w=25\%)}+\frac{\sigma}{E_2(w=25\%)}+\varepsilon_{\eta_{1+}},$$

where:

$$\varepsilon_{\eta_{1+}}=\sigma\cdot\left\lfloor\begin{array}{l} l_{10}\cdot t_{10+}-\frac{1}{r_+}(l_{10}-l_1)\left(1-e^{-r_+t_{10+}}\right)+l_{18}\cdot(t_{18+}-t_{10+}) \\ -\frac{1}{r_+}(l_{18}-l_{10})\left(e^{-r_+t_{10+}}-e^{-r_+t_{18+}}\right)+ \\ l_{25}\cdot(t_+-t_{18+})-\frac{1}{r_+}(l_{25}-l_{18})\left(e^{-r_+t_{18+}}-e^{-r_+t_+}\right) \end{array}\right\rfloor$$

$$\varepsilon_T\left(t=48h^-\right)\cong\frac{\sigma}{E_1(w=1\%)}+\frac{\sigma}{E_2(w=25\%)}+\varepsilon_{\eta_{1+}}+\varepsilon_{\eta_{1-}}+2k\Delta w, \tag{11}$$

where:

$$\varepsilon_{\eta_{1-}}=\sigma\cdot\left\lfloor\begin{array}{l} l_{18}\cdot t_{18-}+\frac{1}{r_-}(l_{25}-l_{18})\left(1-e^{-r_-t_{18-}}\right)+l_{10}\cdot(t_{10-}-t_{18-}) \\ +\frac{1}{r_-}(l_{18}-l_{10})\left(e^{-r_-t_{18-}}-e^{-r_-t_{10-}}\right)+ \\ l_1\cdot(t_--t_{10-})+\frac{1}{r_-}(l_{10}-l_1)\left(e^{-r_-t_{10-}}-e^{-r_-t_-}\right) \end{array}\right\rfloor$$

$$\varepsilon_T\left(t=48h^+\right)\cong\frac{\sigma}{E_2(w=25\%)}+\varepsilon_{\eta_{1+}}+\varepsilon_{\eta_{1-}},$$

$$\varepsilon_T(t=72h)\cong\varepsilon_{\eta_{1+}}+\varepsilon_{\eta_{1-}},$$

where t_{10+}, t_{18+}, t_{10-}, t_{18-} are times determined from formulas (10) after which the sample reaches the moisture content of 10 or 18% during the moistening (+) or drying (-) process, the remaining designations – see Fig. 2 and Tab. 2.

RESULTS AND DISCUSSION

Example results of calculations of ε_T strains, in accordance with formulas similar to those presented above, are presented in Tab. 2. They are collated with the results obtained by Moliński (1984) (ε_D). The differences between the strains calculated theoretically (ε_T) and those determined experimentally (ε_D) are so significant that they cannot be attributed to errors. On the other hand, it can be assumed – as it had already been done befor (Rybarczyk 1973; Rybarczyk and Ganowicz 1974) – that this difference is simply a hygromechanical strain (ε_P) which does not occur in formulas (11) (see formulas (1) and (7)):

$$\varepsilon_P = \varepsilon_D - \varepsilon_T = a\sigma|\Delta w|. \tag{12}$$

Tab. 2: Example comparative collation of strains calculated for the 1→25→1% process in relation of the acting stresses and strains (εD) determined experimentally by Moliński (1984)

Stress σ	Time T	Calculated					Total strain determined experimentally according to (10), ε_D	$\Delta\varepsilon=$ ε_D-ε_T= ε_P
		Components of the total sample strain				Total strain ε_T		
		ε_{E1}	ε_{E2}	$\varepsilon_{\eta 1}$	ε_W			
MPa	H	%						
2.36	0	0.120	0	0	0	0.120	0.08	-0.040
	24	0.241	0.107	0.082	5.09	5.520	6.48	0.960
	48^-	0.120	0.107	0.107	0.02	0.354	2.70	2.346
	48^+	0	0.107	0.107	0.02	0.234	2.33	2.096
	72	0	0	0.107	0.01	0.117	2.05	1.933
3.54	0	0.180	0	0	0	0.180	0.19	-0.010
	24	0.361	0.161	0.122	5.09	5.734	7.84	2.106
	48^-	0.180	0.161	0.160	0.02	0.521	3.74	3.219
	48^+	0	0.161	0.160	0.02	0.341	3.52	3.179
	72	0	0	0.160	0.01	0.170	3.21	3.040
4.72	0	0.241	0	0	0	0.241	0.23	-0.010
	24	0.482	0.215	0.163	5.09	5.950	8.06	2.110
	48^-	0.241	0.215	0.214	0.02	0.690	4.45	3.760
	48^+	0	6.215	0.214	0.02	0.449	4.18	3.731
	72	0	0	0.214	0.01	0.224	3.79	3.566
5.90	0	0.301	0	0	0	0.301	0.32	0.010
	24	0.602	0.268	0.204	5.09	6.164	9.51	3.346
	48^-	0.301	0.268	0.268	0.02	0.857	5.74	4.883
	48^+	0	0.268	0.268	0.02	0.556	5.46	4.904
	72	0	0	0.268	0.01	0.278	5.03	4.752

Therefore, it corroborates – albeit temporarily – the rightness of the overall concept of the model of wood hygromechanical properties. A quenstion however arises how precise will these difference be described with the assistance of the formula (12) resulting from formula (7), i.e. the simplified version of the analysed model. For this purpose, values of the coefficient of hygromechanical coupling (**a**) were determined from experimental data on the basis of the formula resulting from the dependence (12):

$$a = \frac{\varepsilon_D - \varepsilon_T}{\sigma|\Delta w|} \tag{13}$$

and the obtained results are collated in Tab. 3.

Tab. 3: Values of the coefficient of hygromechanical coupling

Specification of the change of sample moisture content	Coefficient of the hygromechanical coupling, **a**				
	MPa^{-1}				
	Stress, σ				
	2.36	3.54	4.72	5.90	Mean
	MPa				
1→10%	0.046	0.047	0.036	0.041	0.042
10→1%	0.026	0.021	0.019	0.021	0.022
1→18%	0.033	0.039	0.034	0.031	0.034
18→1%	0.021	0.020	0.021	0.018	0.020
1→25%	0.017	0.025	0.019	0.024	0.021
25→1%	0.021	0.019	0.017	0.017	0.018
10→1%	0.022	0.021	0.019	0.025	0.022
1→10%	0.011	0.016	0.013	0.014	0.014
18→1%	0.014	0.016	0.015	0.019	0.016
1→18%	0.009	0.012	0.013	0.011	0.011
25→1%	0.021	0.021	0.022	0.021	0.021
1→25%	0.006	0.007	0.007	0.008	0.007

It turns out the results of the application of this theory are quite promising. Moreover, it is evident from the analysis of the data from Tab. 3 that the value of the coefficient of hygromechanical coupling as well as of hygromechanical strains (Hunt 1997, Toratti and Svensson 2000), appears to depend on the range of moisture content changes and, generally speaking, does not depend on the value of the acting external stresses. On the other hand, the absolute values of the '**a**' coefficient depend on the sign of moisture content changes, which is in keeping with the guiding idea of the examined mathematical model (Rybarczyk 1973). Values of the '**a**' coefficient at the increase of moisture content are greater than at the decrease of moisture content within the same interval.

CONCLUSIONS

The performed analysis corroborated the well known fact of significant divergences between the true strains of wood subjected to tension across fibres and changing its moisture content and the sum of theoretically calculated mechanical and moisture content strains of this wood. Consistent results of theoretically calculated and empirically obtained results can be achieved after the application of the hygromechanical model of wood material properties (Rybarczyk 1973, Rybarczyk and Ganowicz 1974) as confirmed by the observed small variability of the values of the coefficients of hygromechanical coupling 'a' (Tab. 3). However, these values indicate that, in order to obtain a more accurate description of wood hygromechanical properties, it is essential to apply a more complex than the applied linear function of hygromechanical coupling F(v).

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