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SHORT NOTICE

A METHOD OF THE DIFFUSION COEFFICIENT DETERMINING DURING HIGH TEMPERATURE ACTING

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ABSTRACT

The method proposed consists of the theoretical and the experimental part. The solution of the diffusion equation, with the constant initial concentration and the appropriate boundary condition forms the base of the theoretical part. The symmetry of the moisture distribution in the cross section of the specimen is supposed. The computer program for the evaluation of the experimental data was designed. The results of the method are the values of the transport characteristics. The experimental part deals with warming up, moisture uptake and measurement. All the parts are combined into one unit. We tested the method during experiments with spruce wood in the tangential direction at 100°C temperature above the saturated aqueous solution of $CaCl_2$.

KEY WORDS: spruce wood, diffusion, method, high temperature, diffusion coefficient, surface emission coefficient, relaxation time

INTRODUCTION

The description of the diffusion of water in wood is based on the 1. Fick's law. The 2. Fick's law or diffusion equation results from the 1. Fick's law and from the law of mass conservation. We solve the diffusion equation under some initial and boundary conditions. The solutions of the diffusion equation form the base of the methods for determining of the diffusion coefficient.

The article of Ping a Liandbai (2003) introduces the diffusion as a part of high temperature drying. These authors cite Miao (2000) who predicted moisture content during wood drying with small samples according Fick's second law. We suppose, that the mechanism of the water movement will be related with the presence of the kind of water in wood (bound or free) and consequently the diffusion is related to bound water. The diffusion coefficient is the measure of the diffusion.

The aim of the article is the description of the method of the diffusion coefficient measurement during high temperature acting at the atmospheric pressure.

Theoretical part of the method is based on the solution of the diffusion equation:

$$\frac{\partial c}{\partial t} = D\frac{\partial^2 c}{\partial x^2} \tag{1}$$

where c is concentration of water in wood, D diffusion coefficient, t time, x spatial coordinate. The mean concentration is used in the methods of the diffusion coefficient measurement, which is related to the solution of the diffusion equation:

$$\bar{c}(t) = \frac{1}{a}\int_0^a c(x,t)dx \tag{2}$$

where a is the half thickness of the specimen. The ratio of mean concentration to the maximum mean concentration is also the ratio of the water taken up to the maximum of this water, which are easier measurable quantities. As we use the constant initial condition, symmetry of the specimen, which is considered to be infinite slab and the boundary condition of the first kind as constant surface moisture during experiment, then for the mean concentration it follows (Požgaj et al. 1993):

$$\frac{\bar{c} - c_0}{c_r - c_0} = 2\sqrt{\frac{Dt}{a^2}}\left[\frac{1}{\sqrt{\pi}} + 2\sum_{n=1}^{\infty}(-1)^n \operatorname{ierfc}\left(\frac{na}{\sqrt{Dt}}\right)\right] = \frac{M_t}{M_r} \tag{3}$$

or

$$\frac{\bar{c} - c_0}{c_r - c_0} = 1 - 8\sum_{n=0}^{\infty}\frac{1}{(2n+1)^2\pi^2}\cdot e^{-\frac{(2n+1)^2\pi^2}{4}\frac{Dt}{a^2}} = \frac{M_t}{M_r} \tag{4}$$

where M_t is the mass of water taken up to the time t and M_r is the mass of water after a long time of moisture uptake. The first equation is used for the diffusion coefficient measurement in the short time as infinite sum could be neglected and the second equation is convenient for the diffusion coefficient measurement in the long times as only the first term of the sum is used. If we solve diffusion equation with the boundary condition of the first kind in the form:

$$\frac{c(a,t) - c_o}{c_r - c_o} = 1 - e^{-\beta t} \tag{5}$$

where β is inverse value of relaxation time then the amount of accepted water is (Babiak et al. 1989):

$$\frac{\bar{c} - c_0}{c_r - c_0} = 1 - 2\sum_{n=0}^{\infty}\frac{e^{-\frac{(2n+1)^2\pi^2}{4}\frac{Dt}{a^2}}}{(2n+1)^2\frac{\pi^2}{4}\left[1 - \frac{(2n+1)^2\pi^2}{4}\frac{D}{\beta a^2}\right]} - \sqrt{\frac{D}{\beta a^2}}\operatorname{tg}\left(\sqrt{\frac{\beta a^2}{D}}\right)e^{-\beta t} = \frac{M_t}{M_r} \tag{6}$$

If we solve the diffusion equation with boundary condition of the third kind in the form:

$$D\left.\frac{\partial c}{\partial x}\right|_{x=a} = \alpha\left(c_r - c(a,t)\right) \tag{7}$$

where α is surface emission coefficient then for the amount of accepted water follow (Požgaj et al. 1993):

$$\frac{\bar{c} - c_0}{c_r - c_0} = 1 - \sum_{n=1}^{\infty}\frac{2Bi^2}{\delta_n^2\left(\delta_n^2 + Bi^2 + Bi\right)}e^{-\delta_n^2\frac{Dt}{a^2}} = \frac{M_t}{M_r} \tag{8}$$

The diffusion coefficient we determine by the least square method. The mean error can be computed as:

$$R = \sqrt{\frac{Q}{n}} \tag{9}$$

where Q is sum of squares and n is the number of the data. The evaluation program searches for the minimum of the sum Q while the diffusion coefficient is the independent quantity. We determined the diffusion coefficient with two significant digits and the sum of squares with four decimal points.

MATERIAL AND METHODS

Experimental part of the method is depicted in figure 1. It consists of three parts.

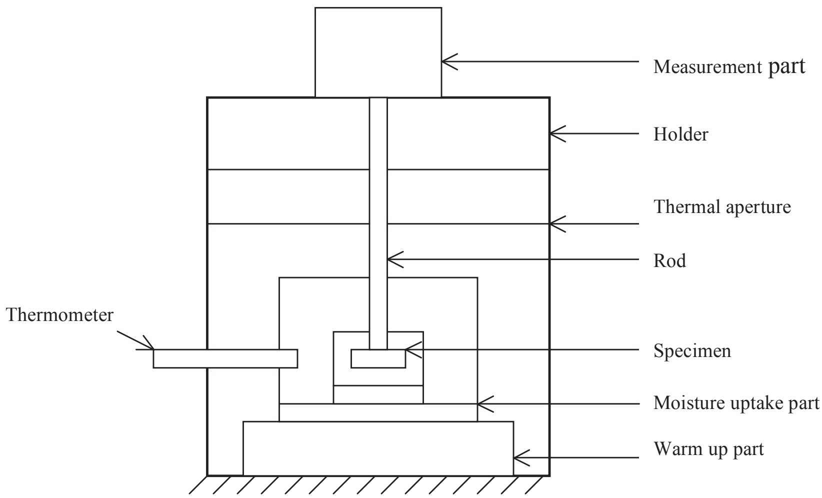

Fig. 1: The schema of the experimental part of the method

The cooker is the base of the warm up part, on which the moisture uptake part is held. The temperature of moisture uptake part is controlled by the cooker's thermostat. The glass container with salt solution is the base of moisture uptake part. The specimen is over the aqueous solution. The specimen is connected with measurement part by the rod. The rod is connected with measurement part – balances (type Kern EW). The computer, which is connected with balances, records the mass of the specimen in time. The whole formation is placed in the holder, which enables the balances to be stable. The heat, from warm up part to the balances, is reduced by thermal apertures.

The basic steps of the measurement are:

1. Drying the specimen to the zero moisture content.
2. Heating up the specimen to the appropriate temperature.
3. Placing the specimen over the saturated solution and recording the moisture uptake.
4. Measurement of the mass and dimensions of the specimens
5. Computation of the diffusion coefficient.

RESULTS AND DISCUSSION

The suggested method was tested on 10 spruce specimens with the dimensions 3x3x1 cm^3 (RxLxT). The specimen took up the moisture over saturated solution $CaCl_2$ in tangential direction. The temperature of the moisture uptake part was 100°C. Figure 2 shows the average data of the moisture in time.

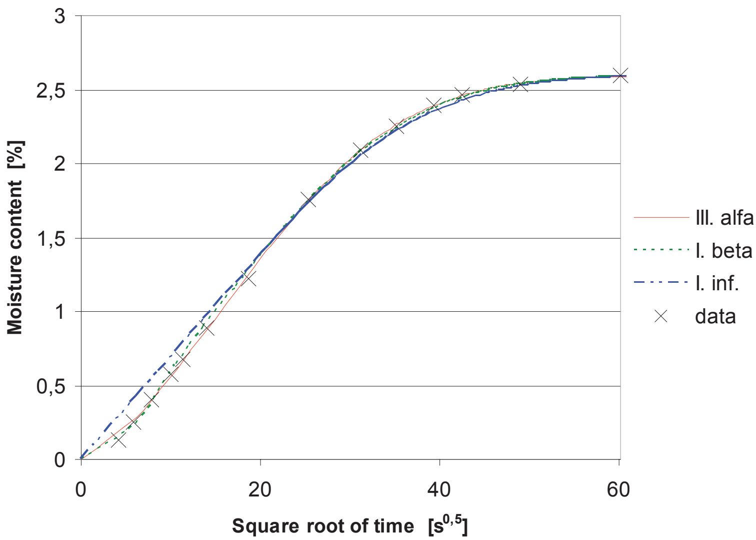

Fig. 2: The dependence of the mean moisture content on square root of time and measured data of the mean moisture content for the mean of the spruce specimen file in tangential direction at 100°C

Tab. 1: The quantities from the equation with the surface emission coefficient

	D $[m^2s^{-1}]$	α $[ms^{-1}]$	Q $[\%^2]$	R [%]	Bi
Average	2,2E-08	3,1E-05	0,0299	0,0436	6,4
St. deviation	6,9E-09	1,5E-05	0,0236	0,0164	1,8

Tab. 2: The quantities from the equation with the relaxation time

	D $[m^2s^{-1}]$	β $[s^{-1}]$	Q $[\%^2]$	R [%]
Average	1,5E-08	3,0E-02	0,0344	0,0430
St. deviation	2,9E-09	2,9E-02	0,0196	0,0175

Tab. 3: The quantities from the equation (3)

	D [m^2s^{-1}]	Q [$\%^2$]	R [%]
Average	1,3E-08	0,1426	0,0982
St. deviation	2,2E-09	0,0679	0,0247

Figure 2 also contains the graphs of functions of average moisture content against square root of time, equations (3), (6), (8). The value of the diffusion coefficient 1,4 10^{-8} m^2s^{-1} results from least square method for equation (3). This value follows also from method, based on the equation (4). The mean error had the value 0,11% for methods based on equations (3) and (4). The values of the sum of squares Q, for these two methods, were different at the sixth decimal place. Therefore we can conclude that the value of mean error is the same for both mentioned methods. The equation (4) is the special case of the equations (6) and (8), if the inverse value of the relaxation time or the surface emission coefficient is sufficiently large. Then the values of the diffusion coefficient are the same. The value of the diffusion coefficient for the method with the relaxation time was $1,5.10^{-8}$ m^2s^{-1} and for the method with the surface emission coefficient $2,1.10^{-8}$ m^2s^{-1}. The surface emission coefficient gains the value $2,1.10^{-5}$ ms^{-1} and inverse value of relaxation time $3,5.10^{-2}$ s^{-1}. The mean error reaches the value 0,03% for the method (6) and 0,01% for the method (8). We can conclude that the method with surface emission coefficient gives the least square error and therefore it is the best description of the diffusion. The values of the diffusion characteristics are not significantly different from values, which were determined from average values of moisture content in time. The equilibrium moisture content after the moisture uptake was 2,60%.

CONCLUSION

In work we designed the method of the measurement of the diffusion coefficient during high temperature uptake. The method was tested in the experiment with the file of spruce specimens at 100°C temperature over the saturated solution of $CaCl_2$ in the tangential direction. The method with the surface emission coefficient yields the least value of the mean error. The method itself is original.

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