STRESS DISTRIBUTION DURING THE DELAMINATION OF LAP JOINTS IN A BENDING TEST

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ABSTRACT

The main goal of this paper was to determine the distribution of stresses in wood lap joint in double-cut samples. During the performed laboratory tests, the authors determined the force destroying the joint. This force, in turn, was used to determine the value and distribution of stresses with the assistance of analytic and numerical methods. Numerical calculation were done using finite element method and brick type elements. Based on analytic model a computer program was written. The distribution of stresses in the glue bond of a lap joint in the course of bending of a double-cut sample shows strong concentration of both normal and tangential stresses at the end of laps. In conclusion it was stated that the mean value of normal stresses in the glue line can be assumed as its peeling strength of double cut wood sample. This value determined numerically amounted to 8.2 MPa.

KEY WORDS: bending, glue line, lap joint, numerical model, peeling stress, shearing stress

INTRODUCTION

When realising an engineer task, attempts are always made to design the construction in such a way that the employed glue joints should carry primarily tangential stresses, while minimising the proportion of the remaining stresses. However, there are situations when a typical operational load of skeleton furniture leads to the development of high normal stresses in the glue bonds of joints as indicated by earlier investigations (Gawroński 2005, 2006). The strength analysis of these construction using analytic or numeric methods requires thorough knowledge of the properties of glue lines subjected to various loads.

In wood science the strength of glue bonds is frequently determined using double-cut samples containing lap joints (Fig. 1a). The principal scheme of loading employed in this kind of investigations is the consequence of stipulations of the EN 205:2003 standard and involves
stretching of the sample along its axis, which allows determining the shear strength of the glue bonds (Fig. 1a).

![Diagram of glue line](image)

Fig. 1: Methods of strength determination of glue bonds employing double-cut samples

The above samples, which were used to determine the strength as well as water- and thermal resistance, were employed in experiments carried out by Boehme 1993, Krystofiak and Proszyk 2001, Proszyk et al. 1997, 1999, while Smardzewski (1998) and Gozdecki (2003) determined the distribution of stresses developing in the glue bond using analytical methods and the method of finite elements.

Comparative investigations of PVAC adhesives were also carried out (Krystofiak 2002, Proszyk et al. 2002, 2005) in which the obtained bonds were subjected to a complex system of shear and peeling stresses resulting from the bending of double-cut samples (Fig. 1b). Until recently, researchers employed the model elaborated by Bock (1951) to analyse peeling stresses in the experimental samples which assumes the linear distribution of normal stresses along the glue line. On the other hand, it is clear from the available literature data dealing with the analysis and strength models of glue lines in various joints (Amijima and Fujii 1987, Edele and Verreman 1992) that one should expect the occurrence of a strongly non-linear distribution of stresses in the discussed case. That is why the model elaborated by Bock is considered to be a significant oversimplification of real conditions and the results of the quoted investigations are applied only to compare properties of various adhesives and changes in the glue line properties caused by the action of unfavourable factors. No papers have been found in literature which would be concerned with the determination of the true distribution of stresses in the glue bond which occur during the bending of double-cut samples and which would allow to utilise the results of laboratory experiments of the discussed joints in construction analysis.

The goal of the study was to determine the distribution of stresses in the glue line during the lap joint bending test applied in numerous laboratory investigations and to verify the compatibility of the Bock’s model employed so far with real conditions. An addition goal of the performed experiments was to determine the delamination strength of the glue line from the PVAC adhesive on the beech wood.

MATERIAL AND METHODS

The achievement of the above-mentioned objectives required carrying out a number of laboratory and theoretical tests and considerations. During the performed laboratory tests, the authors determined the force $F_{\text{max}}$ destroying the joint. This value, in turn, was used to determine the value and distribution of stresses with the assistance of analytic and numerical methods.
Double-cut samples with dimension: $a = 10$ mm, $c = 55$ mm (designations shown in Fig. 1b) and width $b = 20$ mm were used in the experiments. The experimental elements were prepared from beech wood (Fagus sylvatica L.) with the density of 720±40 kg/m$^3$ and moisture content - 12±1%.

The experiments were carried out using the PVAC dispersion adhesive (commercial name – Jowacoll 102.49) together with a hardener 195.40. Both products were manufactured by JOWAT AG in Detmold. The properties of the applied glue are presented in Tab. 1.

**Tab. 1: Properties of the Jowacoll 102.49 glue used in experiments**

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Colour</td>
<td>beige</td>
</tr>
<tr>
<td>Density [g/cm$^3$]</td>
<td>1 ± 0.1</td>
</tr>
<tr>
<td>Viscosity: measured by Brookfield method [mPa*s]</td>
<td>16.000 ± 3.000</td>
</tr>
<tr>
<td>pH</td>
<td>7 ± 0.5</td>
</tr>
<tr>
<td>Minimum film formation temperature of the glue membrane (MTB) [ºC]</td>
<td>5 ± 1</td>
</tr>
<tr>
<td>Conventional content of dry matter [%]</td>
<td>57 ± 2</td>
</tr>
</tbody>
</table>

The glue masses were prepared by weight ($\pm 0.01$g) in accordance with the manufacturer’s recommendation at the ratio of 100 mass parts of glue to 15 mass parts of the hardener. The glueing process was carried out using the following parameters:

- Glue application (one-sided) - 180 g/m$^2$
- Open assembly time - 5 min.
- Pressing - screw clamps
- Pressure - about 0.8 MPa
- Pressing time - 24 hours.

For purposes of laboratory experiments, the total of 10 samples were prepared which were kept in an air-conditioned chamber with constant air temperature of 20±2ºC and relative air humidity of 65±5% for 7 days. Next, the samples were subjected to investigations in accordance with the load design shown in Fig. 1b using, for this purpose, a special test machine MZ and recording the mean breaking force $F_{max}$.

The value of the $F_{max}$ force was used, according to the Bock’s linear model, to determine the maximum normal stress according to the formula below:

$$\sigma_{n,max} = \frac{F_{max}}{ab} + \frac{6F_{max}c}{a^2b}$$  \hspace{1cm} (1)

Then, using the method of finite elements (FEM), the authors elaborated the numerical model of the analysed joint (Fig. 2) which was prepared with the assistance of block elements of brick type. It was assumed in the model that the wooden elements were anisotropic solids for which the material constants were adopted after Wilczyński (1987) for beech wood as follows: $E_L=14010$ MPa, $E_R=1160$ MPa, $E_R=2280$ MPa, $\nu_{L,R}=0.448$ MPa $\nu_{L,R}=0.518$, $G_{L,R}=0.708$, $G_{L,R}=1640$ MPa, $G_{L,R}=1080$ MPa, $G_{L,R}=470$ MPa. On the other hand, the glue bond was treated as an isotropic solid with the Young’s modulus, Poisson’s coefficient and coefficient of rigidity determined by Smardzewski (1998): $E_a=460$ MPa, $\nu_a=0.3$, $G_a=177$ MPa, respectively. For modelling purposes, it was assumed that the thickness of the glue
bond $t$ was 0.1 mm. The value of the external load $F$ was assumed to be equal to the mean breaking force $F_{\text{max}}$ obtained in the course of laboratory experiments. On the basis of the calculations using the finite element method, the distribution of normal $\sigma_z$ and tangential $\tau_{zx}$ stresses was determined in the middle of the glue bond thickness.

![Fig. 2: Numerical model of the considered lap joint](image)

During the subsequent stages of experiments, normal $\sigma_z$ and tangential $\tau_{zx}$ stresses in the glue bond were determined analytically employing for this purpose the mathematical model elaborated by Bigwood and Crocombe (1989). The model referred to a lap joint in which ends of laps were subjected to the action of normal shear forces and bending moments. Adherends were treated as isotropic plates of the rigidity $D$ equal to:

$$D = \frac{Eh^3}{12(1-\nu^2)}.$$  \hspace{1cm} (2)

where: $E$, $\nu$ –Young’s modulus and Poisson’s coefficient of the lap.

The normal and tangential stresses in the glue bond were expressed by differential equations:

$$\frac{d^4 \sigma_z}{dx^4} + 4K_1 \frac{d^2 \sigma_z}{dx^2} = 0,$$  \hspace{1cm} (3)

$$\frac{d^3 \tau_{zx}}{dx^3} - K_2 \frac{d \tau_{zx}}{dx} = 0,$$  \hspace{1cm} (4)

where $K_1$, $K_2$ are constants depending on the elasticity of the applied materials and the geometry of the joint.

The references of the discussed model to the conditions of this problem are presented in Fig. 3 where:
The zero values of forces and moments presented in Fig. 2, as well as the identical rigidity of the laps allowed the authors to simplify some formulas elaborated by Bigwood and Crocombe (1989). Hence, for purposes of this task, the solutions of equations (3) and (4) assume the following form:

\[ V_{11} = V_{22} = -F, \]  

\[ M_{11} = F (c + 0.5 a), \]  

\[ M_{22} = F (c - 0.5 a). \]

The zero values of forces and moments presented in Fig. 2, as well as the identical rigidity of the laps allowed the authors to simplify some formulas elaborated by Bigwood and Crocombe (1989). Hence, for purposes of this task, the solutions of equations (3) and (4) assume the following form:

\[ \sigma_x = A_1 \cos(K_1 x) \cosh(K_1 x) + A_2 \cos(K_1 x) \sinh(K_1 x) + A_3 \sin(K_1 x) \cos(K_1 x) \sinh(K_1 x) \]  

and

\[ \tau_{xz} = B_1 \cosh(K_2 x) + B_2 \sinh(K_2 x) + B_3, \]

where:

\[ K_1 = \sqrt{\frac{E}{2D}}, \]

\[ A_i = \frac{f_i R_1 - f_2 \sinh(K_1 a) \sin(K_1 a) + f_i R_4 - f_1 R_5}{R_5}, \]

\[ A_i = \frac{f_2 R_2 - f_1 \sinh(K_1 a) - f_i R_4 + f_2 \sinh(K_1 a) \sin(K_1 a)}{R_5}, \]

\[ A_i = \frac{f_2 R_2 - f_1 \sinh(K_1 a) - f_i R_4 + f_1 \sinh(K_1 a) \sin(K_1 a)}{R_5}, \]
Bearing in mind the complexity of the relationships (8 – 23), the authors elaborated on their basis a computer program in the C++ language. The program was capable of determining for a set value of the force and elasticity constants of the materials values of normal and tangential stresses along the glue bond beginning with \(x = 0\) and then, each time, increasing the \(x\) value by 0.5 mm until it reached the value of \(x = a\).
RESULTS AND DISCUSSION

Tab. 2 presents the $F_{\text{max}}$ force obtained in the course of the laboratory experiments, wood failure percentage (WFP) and the maximal stress obtained on the basis of the Bock’s model. The presented values do not differ much from the results reported by Krystoński (2002) and Proszyk et al. (2002, 2005).

<table>
<thead>
<tr>
<th>Breaking force $F_{\text{max}}$</th>
<th>mean [N]</th>
<th>91.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard deviation [N]</td>
<td></td>
<td>16.5</td>
</tr>
<tr>
<td>Coefficient of variation [%]</td>
<td></td>
<td>18</td>
</tr>
<tr>
<td>Wood failure percentage WFP [%]</td>
<td></td>
<td>80</td>
</tr>
<tr>
<td>Maximal stress determined by the Bock’s model [MPa]</td>
<td></td>
<td>15.6</td>
</tr>
</tbody>
</table>

On the other hand, Fig. 4 shows the distribution of normal and tangential stresses in the glue bond obtained in the result of numerical calculations. Fig. 5 collates values obtained in the course of numerical and analytic calculations as well as a simplified, linear distribution of stresses obtained on the basis of Bock’s assumptions. Normal and tangential stresses determined with the assistance of both the numerical and analytical methods reveal high extreme values reaching 43.7 and -35.8 MPa for the normal stresses determined using the FEM model. Such high stress values occurred only at the end of laps and at the distance of 0.5 mm, they drop to 13.4 and -14.5 MPa, respectively. Strong stress concentrations in the vicinity of the ends of laps were reported in other studies for various cases of joint loads (Amijima and Fujii 1987, Bigwood and Crocombe 1989, Gozdecki 2003). Also Gawroński (2006), when he analysed the rigidity of wooden tenon joints, reported the occurrence of high normal stresses on small areas in the glue bond which exceeded considerably wood tensile strength across fibres. Simultaneously, in accordance with the EN 205: 2003, the glue bond shear strength is assumed as the mean value of tangential stresses obtained on its entire surface. That is why, using the numerical and analytical models as the basis, the authors decided to determine mean normal and tangential stresses for the analysed case of load of the glue bond. Bearing in mind the two-directional nature of the impacts, the required values were to be determined as means from the absolute values of stresses which occur in the analysed points along the length of the bond. The obtained results are presented in Tab. 3 and, additionally, on diagrams in Fig. 5.
The presented mean value of normal stresses obtained in the course of calculations of the FEM as well as analytical model exceeds by, respectively, 17 and 9% the tensile strength of the beech wood across fibres. This finds its reflection in the experimental part of this research project where the obtained WFP index, which determines the share of the failure occurring in the wood for this joints, was at the level of 80%.

It is apparent from the comparison of the numerical and analytical models as well as the diagrams in Fig. 5 that the stresses determined by both methods are characterised by similar distributions. Considerable discrepancies appear only in the case of maximal normal stresses for which the values obtained analytically is by 37% lower in comparison with the identical ones determined in the course of FEM calculations. On the other hand, the obtained results were much more consistent in the case of mean stresses where the obtained differences amounted to 7 and 8% for normal and tangential stresses, respectively. Extreme tangential stresses determined analytically did not differ from those obtained numerically and the recorded deviations in this case amounted to 3 and 11%, respectively for positive and negative stresses. In the experiments carried out by Edde and Verreman (1992), differences between the results of calculations performed analytically and using the finite element method did not exceed the level of 10%. Discrepancies recorded in this study can be attributed, on the one hand, to not taking into account wood anisotropy in the analytical model and, on the other, to the small length of laps in relation to their thickness which resulted in deviations from the typical character of deformations typical for a plate.

![Diagram](image-url)
CONCLUSIONS

1. The distribution of stresses in the glue bond of a lap joint in the course of bending of a double-cut sample shows strong concentration of both normal and tangential stresses at the end of laps. However, the extreme stresses occur on a small area of the glue bond and cannot be treated as the peeling strength of the glue bond.
2. Mean value of normal stresses in the glue bond can be assumed as its peeling strength. This value determined numerically amounted to 8.2 MPa.
3. Bock’s model of the normal stress distribution in the course of delamination of lap joints is not sufficient, since it differs significantly from real conditions. Therefore, it can be applied only to compare the strength of glue bonds obtained from different adhesives and in different conditions of gluing.
4. The mean value of normal and tangential stresses in the glue bond of a double-cut sample using the adopted scheme of loading can be determined with the assistance of a computer program and analytical model.

REFERENCES

WOOD RESEARCH


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