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MODELLING OF RESPONSE OF AERODYNAMIC EXCITED IDLING CIRCULAR SAW BLADE WITH GEOMETRICAL IMPERFECTION

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ABSTRACT

The basic objective of the present research has been to establish the characteristics of the aerodynamic excitation of an idling circular saw blade with geometrical imperfection i.e. unevenness. A simple model based on the sum of the geometrical imperfections of the saw blade plate and the associated travelling waves is proposed. Presented is the solution of the wave equation which considers the geometrical unevenness of the circular saw blade plate. Geometrical imperfection of circular saw blade has no direct influence on the excitation mechanism of an idling saw blade. The consequence of the time-variable turbulent airflow is the occurrence of variable air pressure along the tool surfaces, which constitutes the aerodynamic excitation of the saw blade and can be directly observed only by a stationary microphone. The aerodynamically excited natural modes of the lateral vibrations of a rotating saw blade can be observed directly with a stationary inductive displacement sensor but only indirectly through the frequencies of forward and backward travelling waves.

KEY WORDS: vibration, aerodynamic excitation, rotating circular saw blade, travelling wave

INTRODUCTION

A large number of authors have dealt with the issue of stability or lateral vibration of circular saw blades in the past; however, the most important work was undoubtedly accomplished by Southwell, when he explained the effect of the speed of rotation on the natural frequencies of the tool (Pahlitzsch and Rowinski 1966, 1967; Pahlitzsch and Friebe 1966). Of great importance are also researches carried out by Mote and co-workers (Mote 1964, Mote and Nieh 1973, Mote and Szymani 1977, Leu and Mote 1984, Yu and Mote 1987). Despite the fact that the circular saw blade is one of the oldest woodworking tools, both its construction and manufacture as such are still rather problematic for the producers of woodworking tools of today. The problematic nature of the construction arises from the fact that the characteristic construction of circular saw blades is the consequence of a compromise between the technological or purpose-related requirements and the stability – rigidity requirements. Above all, the manufacture of circular saw blades is problematic from the aspect of internal stresses in the saw plate of the tool that is made of rolled sheet metal, i.e. of a material, which already has a definite but usually unknown stress state incorporated. The transversal or lateral time-variable displacements of the saw plate of an axially clamped idling circular saw blade represent a conventional forced vibration of a deformable structure. Since a forced vibration is known to represent a time-variable response of the system subjected to exciting forces, it may be concluded that the lateral vibrations of idling circular saw blades are solely the consequence of the action of the time-variable forces acting in the direction of displacements, that is transversely to the plane of the saw plate. If the lateral displacement of the saw plate is w then the plate equation may be written in a polar coordinate system attached to the disc according to Hutton (1991) and slightly modified as

$$D\nabla^4 w + L_1(w) + L_2(w) + \rho h\ddot{w} = F(r,t) \tag{1}$$

The terms on the left side of the equation represent the plate bending stiffness, the in-plane tensioning, in-plane rotational stresses and inertial stresses, respectively. The term on the right side represents the excitation forces, which are in our case the result of interaction between the tool and the flows of the surrounding moving air. Assuming that the lateral vibration of rotated circular saw blades is a sum of a larger number of modal in space stationary vibration modes having characteristic form, frequency and amplitude, the approximating solution can be written as (Yu and Mote 1987, Tian and Hutton 1999)

$$w(r,\theta,t) = \sum_{n=0}^{N}\sum_{m=0}^{M}\{R_{mn}(r)[C_{mn}(t)\cos(n\theta) + S_{mn}(t)\sin(n\theta)]\}, \tag{2}$$

where $R_{mn}(r)$ is a linear combination of Bessel functions and known natural frequency parameters. $C_{mn}(t)$ and $S_{mn}(t)$ are unknown functions to be determined. m and n are the numbers of nodal circles and nodal diameters, respectively.

The proposed model based on wave equation

In the case when the coordinate system of the observer is fixed in space and not attached to the rotating saw blade, we can obtain the expression for the time dependent lateral displacement of the rotating circular saw blade by solving the wave equation which may be written in a polar coordinate system as

$$\frac{1}{c^2}\frac{\partial^2 w}{\partial t^2} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 w}{\partial \varphi^2} + \frac{\partial^2 w}{\partial z^2} \tag{3}$$

were c is wave propagation velocity. Because of the constant vibration amplitude over the circular saw blade thickness, the last term on the right side of equation is equal to zero. If we observe the lateral vibration of a rotated saw blade at constant radius, the first term at the right side of equation becomes irrelevant and the wave equation may be rewritten as

$$\frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} = \frac{1}{r^2}\frac{\partial^2 u}{\partial \varphi^2} \tag{4}$$

By taking the radius as unity, the equation 4 may be rewritten in a form of the one-dimensional wave equation

$$\frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial \varphi^2} \tag{5}$$

Changing the linear wave propagating velocity c with angular wave propagating velocity ω, the so called d`Alembert solution of equation 5 may be written as

$$w(\varphi, t) = \frac{1}{2}\left[F(\varphi + \omega t) + F(\varphi - \omega t)\right] + \frac{1}{2\omega}\int_{\varphi-\omega t}^{\varphi+\omega t} G(s)ds \tag{6}$$

Considering the initial conditions

$$\frac{\partial w(\varphi,0)}{\partial t} = G(\varphi) = 0$$

$$w(\varphi,0) = F(\varphi) = A\sin(n\varphi)$$

the equation 6 may be rewritten as

$$w(\varphi, t) = \frac{1}{2}A\left[\sin(n\varphi + \omega t) + \sin(n\varphi - \omega t)\right] \tag{7}$$

where A and n are vibration amplitude and vibration mode i.e. number of nodal diameters respectively. The solution represents the sum of two travelling waves propagating in different directions. Taking into account also the rotational speed of the saw blade Ω, equation 7 may be written in the case of a perfectly flat circular saw blade as

$$w(\varphi, t) = B\left[\sin(n\Omega t + \omega t) + \sin(n\Omega t - \omega t)\right] \tag{8}$$

The first wave, named as a forward travelling wave, propagates in the direction of rotation, while the second wave, named as a backward travelling wave, propagates in opposite direction of the saw blade rotation. If the rotated saw blade is not perfectly flat, the unevenness of the saw blade plate also has to be considered in equation 8. Marking the unevenness of the saw blade as H (φ), the final solution of the wave equation for a rotated saw blade with known unevenness may be written as

$$w(\varphi, t) = H(\Omega t) + B\left[\sin(n\Omega t + \omega t) + \sin(n\Omega t - \omega t)\right] \tag{9}$$

MATERIAL AND METHODS

The laboratory part of the research was carried out on the purpose-built experimental device (Fig. 1). A circular saw blade with 60 teeth made of carbide metal and with four thermal expansion slots was clamped directly to the shaft of an electric motor with two flanges of 89 mm diameter each. The nominal diameter of the circular saw blade was 250 mm. The thickness of the saw blade plate was 2.7 mm. For driving the tool, a two-pole three-phase asynchronous electric motor with a rated output of 4 kW was used. The electric motor was connected to a Fuji Electric three-phase digital frequency converter FRENIC 5000G11S-EN with the output frequency in the range from 0 to 400 Hz. An inductive proximity displacement sensor with a proportional voltage output and a Bruel&Kjær 4939 microphone monitored the idling circular saw blade. The outputs of the both transducers were connected to a National Instruments AT-MIO-16E-1 data acquisition board. The measurements were supervised with LabView 7.1 software.

Natural frequencies of a stationary axially clamped circular saw blade were obtained by frequency response analysis. The coefficients of dynamic rigidity were determined on the basis of the analysis of the acoustic response of the pulse-excited rotating circular saw blade.

The correlation between the rate of revolutions of the saw blade and its natural frequencies, the respective frequencies of the forward and backward travelling waves were also calculated for individual nodal vibration modes. The relevant frequencies of the vibration modes were determined indirectly on from the measured frequencies of the forward and backward travelling waves. The natural frequencies of the circular saw blades and the nodal vibration modes were checked by the finite element method. For this purpose, the finite element method computer program ALGOR was used.

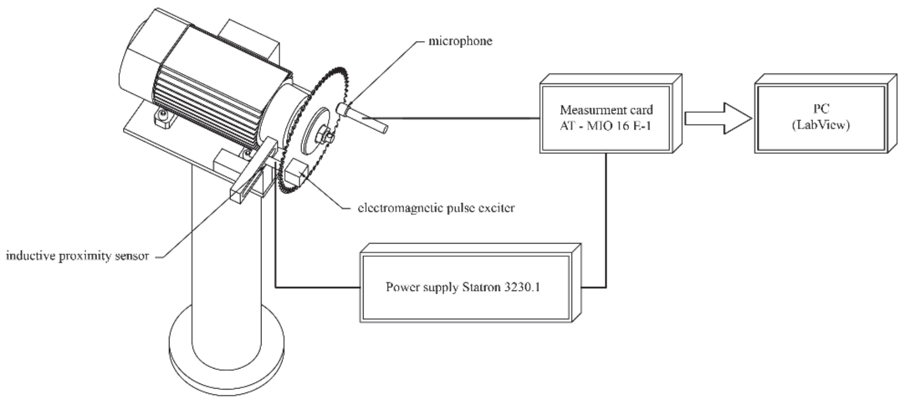

Fig.1: Experimental system

RESULTS AND DISCUSSION

The unevenness or lateral deformation of an axially clamped circular saw blade $H(\varphi)$ was measured with a fixed inductive proximity sensor with proportional voltage output during the very low rotation frequency of the saw blade. It is evident that the lateral deformation exceeds the usually allowed value of 0.1 mm (Fig. 2). The frequencies of characteristic natural vibration modes were determined by the frequency response of a pulse-excited stationary axially clamped circular saw blade within the frequency range from 100 to 2100 Hz (Fig. 3). The frequencies characteristic of individual modes of the natural vibrations of a circular saw blade can be seen well. The vibration mode designations are given in brackets, wherein the first number stands for the number of nodal circles and the second number for the number of nodal diameters. The letters s and c at the second number denote sine or cosine vibration mode. The frequency range was selected in such a way that all relevant natural vibration modes can be seen. Within the range of low frequencies of rotation, the excitation of natural vibration modes did not occur. The reason for this lies most probably in too low pressure differences that develop during the aerodynamic excitation of the tool, which in turn also results in too small changes in lateral exciting forces. By increasing the frequency of rotation, the situation, however, obviously changes to the extent that all conditions for the excitation of natural vibration modes of the circular saw blade or resonant states are fulfilled. The transition from one resonant condition into another is abrupt. With the increasing frequency of rotation the range in which the aerodynamic excitation of natural manners of vibration occurs also increases as a rule. Natural vibration modes were also checked by the finite element method.

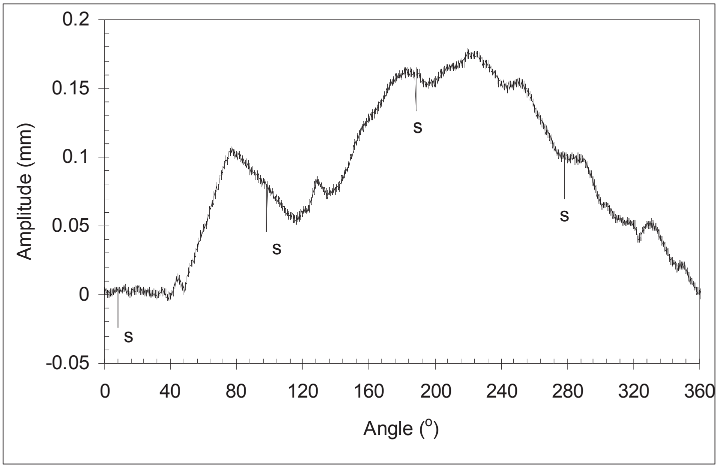

Fig. 2: Geometrical imperfection – unevenness of circular saw blade used in investigation (s – thermal expansion slot)

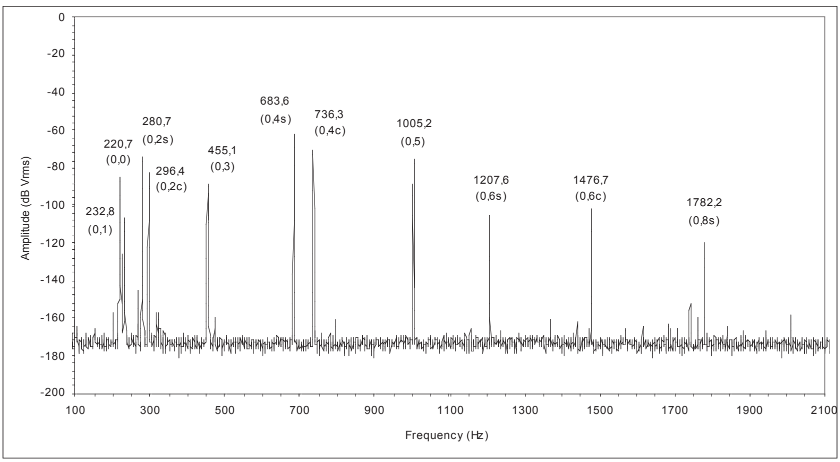

Fig. 3: Frequency response of clamped standing still circular saw blade between 100 and 2100 Hz

In the case of an idling circular saw blade, the frequency of aerodynamic excitation is distributed over a narrower and wider frequency range. The width of the relevant excitation frequency range increases with the increasing frequency of rotation. The correlation between the natural frequency of a given vibration mode and frequency of rotation defines the coefficient of dynamic rigidity k_n. In our case, the coefficients of dynamic rigidity of individual natural vibration modes were between 2.18 and 5.56, which mean that the changes of natural frequencies due to tool rotation within the rate of rotational frequency subject to the research are rather small.

The comparisons between the time dependent measured and modelled lateral displacement of the rotating saw blade are shown in Fig. 4 and 5. We used equation 9 to model the lateral displacement of a rotating saw blade in resonance modes with 4 and 5 nodal diameters. The measured signal was recorded with an inductive proximity sensor with proportional DC voltage output directed perpendicular to the saw blade plane and placed at the peripheral part of the rotating saw blade. Because the modelled values correspond to those measured very well, the suggested simple model based on the sum of geometrical imperfections of saw blade plate $H(\varphi)$ and travelling waves is in general approved. The geometrical imperfection is the main source of lateral displacement of an idling saw blade. From the spectra obtained by the FFT of the measured and modelled lateral displacement of the rotating saw blade in resonance mode (0,4) shown in Fig. 6 and 7, the two frequency peaks are distinguished. According to the Mote and Szymani (1977), the peaks represent the forward and backward travelling waves.

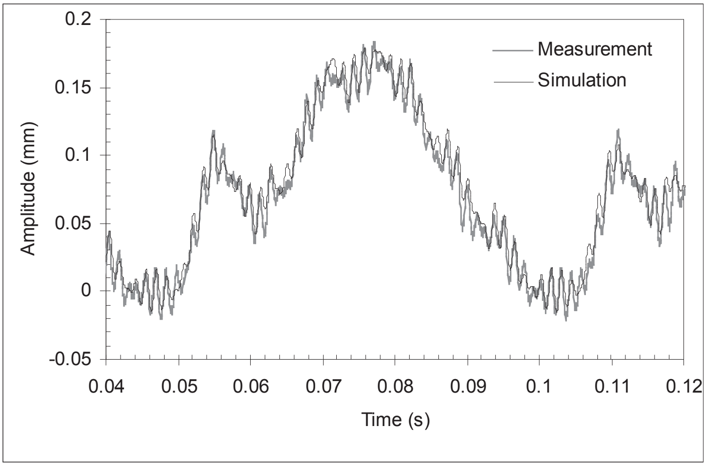

Fig. 4: Comparison between measured and modelled time dependent lateral vibrations of rotated circular saw blade in resonance mode with four nodal diameters (0,4) at rotational frequency of 18 Hz

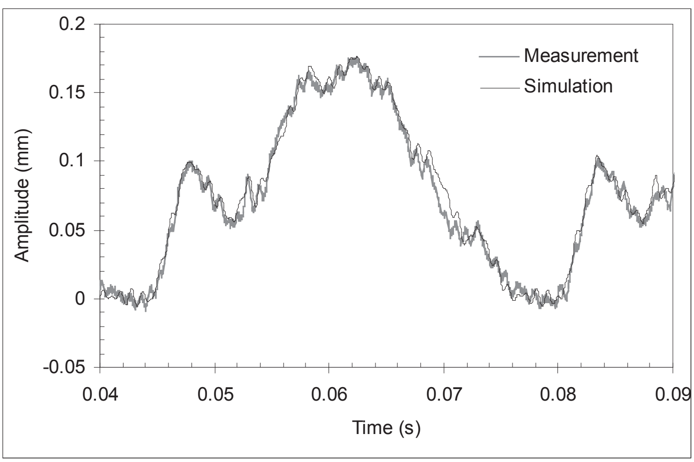

Fig. 5: Comparison between measured and modelled time dependent lateral vibrations of rotating circular saw blade in resonance mode with five nodal diameters (0,5) at rotational frequency of 28 Hz

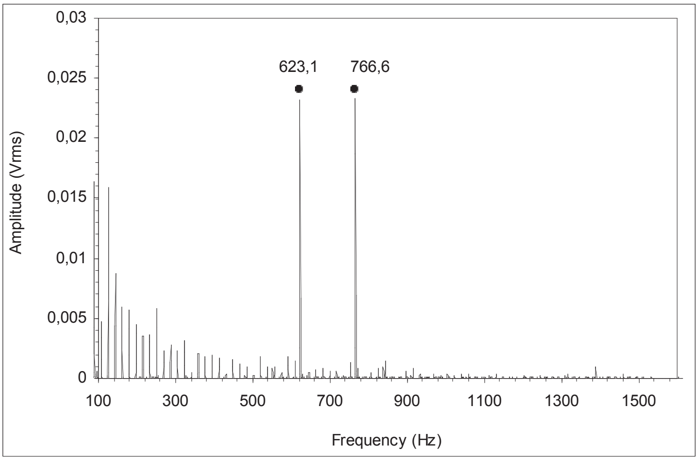

Fig. 6: Frequency spectrum of resonance vibration of rotated saw blade in mode (0,4) at rotational frequency of 18 Hz; measured with inductive proximity sensor

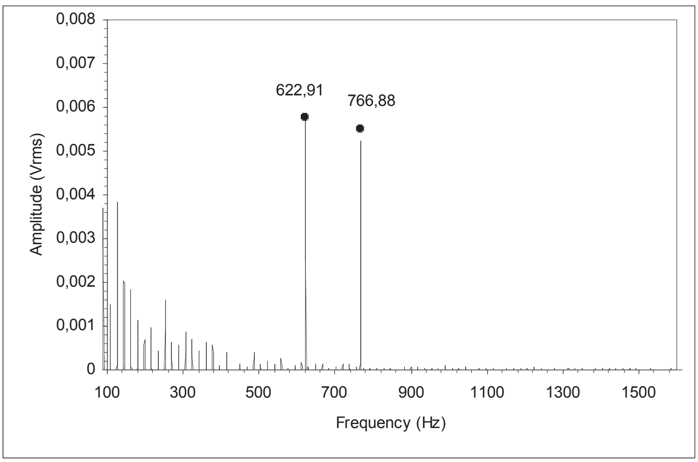

Fig. 7: Simulated frequency spectrum of resonance vibration of rotated saw blade in mode (0,4) at rotational frequency of 18 Hz

In spite of the resonance, the excitation frequency or natural response frequency of the circular saw blade can not be observed in the frequency spectre of measured lateral displacement of the rotating saw blade (Fig. 6). The reason for that lies in the fact that the observing system, i.e., the inductive proximity sensor was at a standstill.

In addition to the inductive proximity sensor, we also used a microphone to monitor the aerodynamic excitation and response of an idling circular saw blade with geometrical imperfection. The comparison of the spectra shown on Fig. 6 and 8 shows that the frequency aerodynamic excitation (694.6 Hz on Fig. 8) also occurs in the frequency pattern of the signal obtained with the microphone. During the aerodynamic excitation of the rotating saw blade, the time-variable turbulent airflow of variable pressure which occurs along the tooth surfaces can be directly observed only by the stationary microphone. If the frequency of the varying excitation air pressure is in the proximity of one of the natural frequencies of the saw blade, resonance or increase in the amplitude of the lateral vibration of saw blade can occur. In resonance, the frequency of vortex separation coincides with a natural frequency of the rotating saw blade and the whistling noise occurs. In our case the conditions for excitation of the natural vibration mode of the rotating saw blade with 4 nodal diameters were fulfilled.

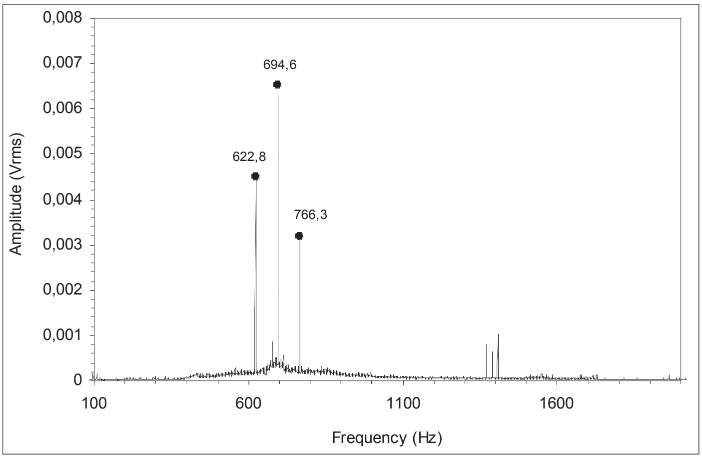

Fig. 8: Frequency spectrum of resonance vibration of rotated saw blade in mode (0,4) at rotational frequency of 18 Hz; measured with microphone

CONCLUSIONS

A simple model based on the sum of geometrical imperfections of saw blade plate $H(\varphi)$ and travelling waves is proposed. Because the modelled values correspond with the measured ones very well, the suggested simple model is in general approved. Beside a geometrical imperfection, for a proper modelling of the response of an aerodynamically excited rotating saw blade, the correlation between the natural frequency of a given vibration mode and frequency of rotation i.e. the coefficient of dynamic rigidity must be known. Geometrical imperfection of circular saw blade has no direct influence on the excitation mechanism of an idling saw blade. From the spectra obtained by the FFT of the measured and modelled lateral displacement of the rotating saw blade in resonance mode, the two frequency peaks are distinguished. The peaks represent the forward and backward travelling waves. Despite the resonance, the excitation frequency or natural response frequency of the circular saw blade cannot be observed in the frequency spectre of measured lateral displacement of a rotating saw blade. This is because the observing system, i.e., the inductive proximity sensor was at a standstill. The natural frequency of the lateral vibration mode of the rotating tool can be measured solely with the measuring system that rotates together with the circular saw blade. During the aerodynamic excitation of the rotating saw blade natural modes, the time-variable turbulent airflow of variable pressure which occurs along the tooth surfaces can be directly observed only by the stationary microphone.

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