

## 3D ANALYSIS OF STRESS DISTRIBUTION IN VENER PLYWOOD UNDER BENDING LOAD

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### ABSTRACT

In the paper, an analysis is conducted of all six stress components and all six strain components, for the purpose of determining the importance of individual stress components that occur in veneer plywood under bending load. A 3D analysis was conducted using the finite element method, statics module - linear elastic theory. Two numerical models were created in line with the test method (EN 310:1993) for determining the bending properties of veneer plywood. One model was created such that the load direction is parallel to the face grain direction, and the other when the load direction is perpendicular to the face grain direction. The results shown include the distribution of stress along the plywood surface, as well as the stress distribution in individual layers, i.e. by plywood thickness.

The stress results relate to the direction of the axis in the global Cartesian coordinate system. The highest values are seen with normal stress, in particular the component  $\sigma_x$  which has a direction parallel to the load direction. In shear strain, the highest values are seen in the component  $\tau_{xz}$ . The value of  $\sigma_x$  varies stepwise linearly, through the thickness of the veneer plywood, which is the direct result of the cross-orientation of individual layers. Unlike  $\sigma_x$ , the value of the strain component  $\tau_{xz}$  continually increases linearly from the lower to the upper layer. The component  $\tau_{xz}$  also characterises the greatest increase in shear strain values. Though the remaining components of normal strain  $\sigma_y$  and  $\sigma_z$ , and shear strain  $\tau_{xy}$  and  $\tau_{yz}$  are substantially lower, they are also very important in analysing the stress of plywood, as they cause significant strain in the veneer.

KEY WORDS: plywood, bending, stress, finite element method, three dimensions

### INTRODUCTION

A classical analysis of stress of the panel and laminates is primarily based on a two-dimensional, and rarely on three-dimensional analysis. In two-dimensional stress analyses, individual stress components are most commonly ignored, particularly those thought to have a minimal influence on the final outcome. Ignoring individual stress components has great practical value, as this then simplifies a very complicated analytical solution to the problem. On the other hand, the development of numerical models (finite element method) and the application of computers has allowed us to achieve comprehensive results with a minimal share of individual approximations.

Numerous published papers analyzing plywood panels are predominantly based on two-dimensional studies (Gerrard 1987, Booth 1990, Lang et al. 2003, Brezović et al. 2003, Kljak et al. 2006), in which the stress in the direction of the thickness of the veneer or plywood panel is ignored. If the structural construction of the plywood panel is so considered, then it is evident that the thickness of individual layers and their orientation, play an important role in establishing the mechanical properties of the veneer plywood.

Under bending load, stress, i.e. both normal and shear stress, is transferred through the thickness of the panel and the thickness of each individual layer in the transversal direction. Considering the type of load, it can be assumed that the amount of stress in the transversal direction will be somewhat lower in value, but certainly not absolutely negligible. In addition to stress in the transversal direction, it is very important to simultaneously consider stress in the longitudinal and lateral directions, particularly where the analysis is based on the orthotropic properties of the material, i.e. veneer. Therefore, the objective of this study is to conduct a detailed three-dimensional analysis of stress in individual layers of the veneer panel under bending load.

## MATERIAL AND METHODS

Numerical models and empirically derived values were used in the stress analysis. The values of the mechanical properties determined empirically were used to create the numerical models. The control panel was a multi-layer birch plywood with a nominal thickness of 15 mm (14.86 mm) – EN 325:1993. The plywood panel had a total of 11 cross-oriented layers. The thickness of the outer layers was 1.2 mm, while all inner layers were 1.4 mm thick. Individual physical and mechanical properties were determined according to European standards. Panel density totalled  $690 \text{ kg/m}^3$  (EN 323 :1993) and moisture content was 8.6% (EN 322:1993). Bending strength and the modulus of elasticity of the bending strength were determined using the three point load method (EN 310 :1993). Bending strength parallel to the face grain direction was ( $f_{m\parallel} = 103 \text{ MPa}$ ), bending strength perpendicular to the face grain direction was ( $f_{m\perp} = 73 \text{ MPa}$ ), the modulus of elasticity of the bending strength parallel ( $E_{m\parallel} = 12046 \text{ MPa}$ ), and perpendicular ( $E_{m\perp} = 8611 \text{ MPa}$ ).

The static module – linear elastic theory of the COSMOS/M software package was used to create the numerical models. In three dimensions, the orthotropic strain-stress has the following form (SRAC 2001):

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix} = \begin{bmatrix} 1/E_x & -\nu_{xy}/E_x & -\nu_{xz}/E_x & 0 & 0 & 0 \\ -\nu_{xy}/E_x & 1/E_y & -\nu_{yz}/E_y & 0 & 0 & 0 \\ -\nu_{xz}/E_x & -\nu_{yz}/E_y & 1/E_z & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/G_{xy} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/G_{yz} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/G_{zx} \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix} \quad (1)$$

Furthermore, in three dimensions, the orthotropic symmetry conditions dictate:

$$\frac{\nu_{xy}}{\nu_{yx}} = \frac{E_x}{E_y}, \frac{\nu_{yz}}{\nu_{zy}} = \frac{E_y}{E_z}, \frac{\nu_{zx}}{\nu_{xz}} = \frac{E_z}{E_x} \quad (2)$$

The orthotropic properties defined above were used by the multi-layer three-dimensional solid element which was selected as the basic element. Integration type of element was hybrid -

displacement and stress-based (mixed) formulation. Number of layers in the element was 11. A stress was calculated in the global Cartesian system. Each layer has defined orthotropic properties as follows: modulus of elasticity:  $E_x = 16300$  MPa,  $E_y = 620$  MPa,  $E_z = 1110$  MPa; modulus of rigidity:  $G_{xy} = 910$  MPa,  $G_{yz} = 190$  MPa,  $G_{xz} = 1180$  MPa; Poisson's ratio:  $\nu_{xy} = 0.43$ ,  $\nu_{yz} = 0.38$ ,  $\nu_{xz} = 0.49$  (Dinwoodie 1981). In the Cartesian coordinate system, the direction of the axis also marks the three main wood directions, i.e. in the direction of the  $x$ ,  $y$ ,  $z$  axes, marking the longitudinal, tangential and radial direction of the wood. Two models were created: the first for when the load acts parallel to the face grain direction (model-parallel) and the second when the load acts perpendicular to the face grain direction (model-perpendicular). The load (732 N) was equal in both models, i.e. 40% of the maximum force in the perpendicular direction (empirically derived) so as not to exceed the limit of elastic stress of the material. A total of 216 elements were defined for creation the finite element mesh. Fig. 1 shows the distribution of supports and the force on the numerical model, set up according to norm EN 310: 1993.

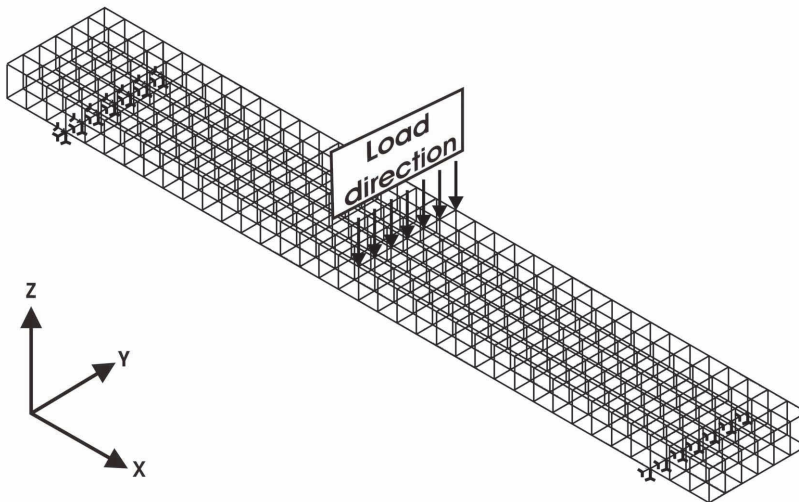


Fig. 1: FEM model of the bending load of plywood

## RESULTS AND DISSCUSION

3D analysis of the stress distribution in all layers of the veneer plywood is a very complex problem that can be approached in several ways. Therefore, the analysis of the results is based on monitoring the critical values of stress under bending load. Considering that under bending load, critical stress appears directly under the central point load, particularly in the first, i.e. lower layer, the results outlined below relate to this area.

Fig. 2 show all three components of normal stress, while Fig. 3 show all three components of shear stress (shown by node). All figures show the distribution of individual stress components and their values in the first, i.e. bottom layer of the veneer plywood.

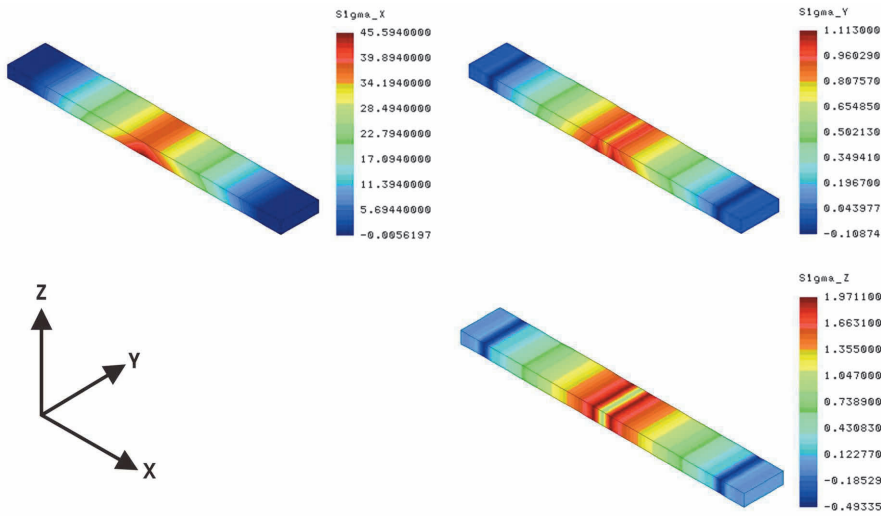


Fig. 2: Distribution of stress components  $\sigma_x$ ,  $\sigma_y$ ,  $\sigma_z$  in the first layer of veneer plywood

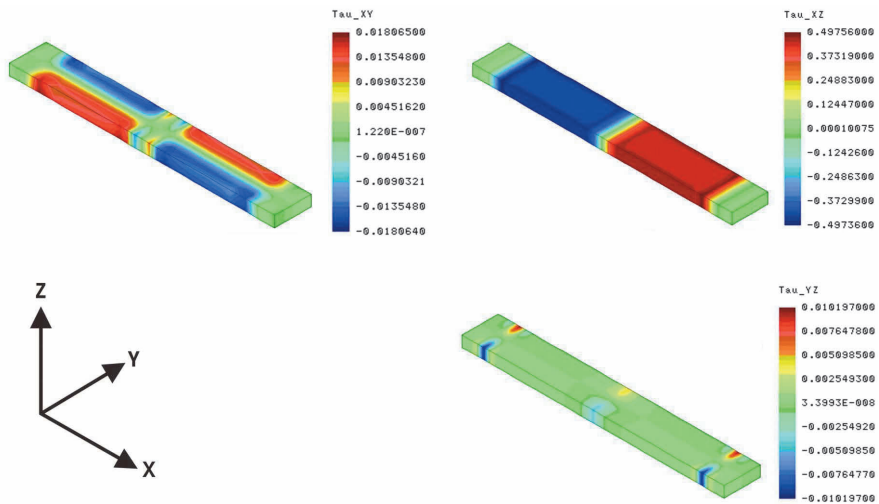


Fig. 3: Distribution of stress components  $\tau_{xy}$ ,  $\tau_{xz}$ ,  $\tau_{yz}$  in the first layer of veneer plywood

The figures shows that the highest normal stress appears under the central point load and gradually decreases towards the free edges of the plywood, where it virtually disappears. Exceptionally high stress values are seen in the component  $\sigma_x$  while the components  $\sigma_y$  and  $\sigma_z$  show markedly lower values. The highest stress is in component  $\sigma_x$  as its direction is parallel to the load direction, which is also the longitudinal direction of the wood, which has the highest modulus of elasticity and strength. Therefore, layers with such orientation take on the highest share of the stress. The low values of the component  $\sigma_y$  are the result of a lower modulus of elasticity in the tangential direction of wood, also

because its direction is perpendicular to the load direction. The stress component  $\sigma_z$  arises from the transfer of the load from the top layer of the plywood to the bottom layer, i.e. transfer of load in the transversal direction of the plywood. Unlike normal stress, the highest shear stress is distributed along the edges of the plywood, with the highest stress in component  $\tau_{xz}$ , and substantially lower stress in components  $\tau_{xy}$  and  $\tau_{yz}$ . In shear stress, we can apply similar dependence as in normal stress. The highest stress was in  $\tau_{xz}$  as its direction is also parallel to the load direction, considering that the following is true for orthotropic materials:  $\tau_{xz} = \tau_{zx}$ .

Fig. 2 and Fig. 3 show the values of individual stress components in the first layer of the veneer plywood. However, these values change with the distance of individual layers from the panel's neutral axis. Tab. 1 gives the values (by element) of all six stress components that arise in the centre of each individual layer of the veneer plywood. The table gives the result of both numerical models, i.e. the distribution of stress when the load direction is parallel to the face grain direction (model – parallel) and when it is perpendicular (model – perpendicular). This presentation of stress clearly shows the influence of individual stress components on the magnitude of the overall stress that appears under bending load.

Tab. 1: Stress components in all plywood layers

Layer	$\sigma_x$ MPa	$\sigma_y$ MPa	$\sigma_z$ MPa	$\tau_{xy}$ MPa	$\tau_{xz}$ MPa	$\tau_{yz}$ MPa
model-parallel						
11	-41.7700	-1.1480	-2.2950	0.0444	-0.6497	0.0017
10	-1.4660	1.3400	-1.0000	0.0431	-0.6112	0.0018
9	-25.3700	-0.7346	-1.5630	0.0417	-0.5696	0.0019
8	-0.7789	0.7037	-0.7124	0.0403	-0.5280	0.0021
7	-8.3750	-0.3064	-0.8027	0.0389	-0.4865	0.0022
6	-0.0914	0.0669	-0.4243	0.0375	-0.4449	0.0023
5	8.6240	0.1218	-0.0428	0.0361	-0.4034	0.0025
4	0.5960	-0.5699	-0.1362	0.0347	-0.3618	0.0026
3	25.6200	0.5501	0.7172	0.0334	-0.3203	0.0027
2	1.2830	-1.2070	0.1519	0.0320	-0.2787	0.0029
1	42.0200	0.9630	1.4500	0.0307	-0.2401	0.0030
model-perpendicular						
11	-2.6390	1.0580	-1.5410	0.0447	-0.7719	0.0014
10	-50.2700	-1.4080	-2.7000	0.0433	-0.7281	0.0015
9	-1.6420	0.6622	-1.1060	0.0419	-0.6810	0.0016
8	-25.0600	-0.7504	-1.5630	0.0404	-0.6339	0.0017
7	-0.6086	0.2522	-0.6542	0.0390	-0.5868	0.0019
6	0.1525	-0.0932	-0.4256	0.0375	-0.5397	0.0020
5	0.4253	-0.1579	-0.2026	0.0361	-0.4925	0.0021
4	25.3600	0.5640	0.7115	0.0346	-0.4454	0.0022
3	1.4590	-0.5680	0.2489	0.0331	-0.3983	0.0023
2	50.5800	1.2210	1.8490	0.0317	-0.3512	0.0025
1	2.4560	-0.9634	0.6843	0.0303	-0.3074	0.0026
Layer numbering is from the bottom to the top layer.						

Tab. 2: Strain components in all plywood layers

Layer	$\varepsilon_x$	$\varepsilon_y$	$\varepsilon_z$	$\gamma_x$	$\gamma_{xz}$	$\gamma_{yz}$
model-parallel						
11	-2.46E-03	1.82E-04	-3.47E-04	4.88E-05	4.81E-06	-2.08E-03
10	-2.00E-03	1.51E-04	-3.47E-04	4.74E-05	5.18E-06	-1.96E-03
9	-1.49E-03	1.18E-04	-3.47E-04	4.58E-05	5.58E-06	-1.82E-03
8	-9.86E-04	8.51E-05	-3.47E-04	4.43E-05	5.98E-06	-1.69E-03
7	-4.82E-04	5.22E-05	-3.47E-04	4.28E-05	6.38E-06	-1.56E-03
6	2.28E-05	1.93E-05	-3.47E-04	4.12E-05	6.78E-06	-1.43E-03
5	5.27E-04	-1.37E-05	-3.47E-04	3.97E-05	7.19E-06	-1.29E-03
4	1.03E-03	-4.66E-05	-3.47E-04	3.82E-05	7.59E-06	-1.16E-03
3	1.54E-03	-7.95E-05	-3.47E-04	3.67E-05	7.99E-06	-1.03E-03
2	2.04E-03	-1.13E-04	-3.47E-04	3.51E-05	8.39E-06	-8.93E-04
1	2.51E-03	-1.43E-04	-3.47E-04	3.37E-05	8.76E-06	-7.69E-04
model-perpendicular						
11	-3.66E-03	1.81E-04	-3.50E-04	4.91E-05	4.43E-06	-2.25E-03
10	-2.97E-03	1.50E-04	-3.50E-04	4.76E-05	4.78E-06	-2.12E-03
9	-2.22E-03	1.17E-04	-3.50E-04	4.60E-05	5.17E-06	-1.98E-03
8	-1.47E-03	8.42E-05	-3.50E-04	4.44E-05	5.55E-06	-1.84E-03
7	-7.23E-04	5.12E-05	-3.50E-04	4.28E-05	5.93E-06	-1.71E-03
6	2.46E-05	1.82E-05	-3.50E-04	4.12E-05	6.32E-06	-1.57E-03
5	7.72E-04	-1.48E-05	-3.50E-04	3.96E-05	6.70E-06	-1.43E-03
4	1.52E-03	-4.78E-05	-3.50E-04	3.80E-05	7.08E-06	-1.30E-03
3	2.27E-03	-8.08E-05	-3.50E-04	3.64E-05	7.47E-06	-1.16E-03
2	3.02E-03	-1.14E-04	-3.50E-04	3.48E-05	7.85E-06	-1.02E-03
1	3.71E-03	-1.45E-04	-3.50E-04	3.33E-05	8.21E-06	-8.94E-04
Layer numbering is from the bottom to the top layer.						

By monitoring stress in individual layers by panel thickness, it is evident that the values of normal stress  $\sigma_x$  change stepwise linearly, which is the direct consequence of the cross-orientation of the layers. Unlike this, the normal stress components  $\sigma_y$  and  $\sigma_z$  have significantly equalized values. Considering the plywood thickness (direction  $z$ ), all the layers have an equal orientation, i.e. radial direction of the wood, and therefore stress  $\sigma_z$  is of equal value through the individual layers. The direction of component  $\sigma_y$  is perpendicular to the load direction, following which lower and equalized stress values appear.

In a comparison of normal stress  $\sigma_x$ , it is evident that a higher stress appears in layer 2,  $\sigma_x = 50.2$  MPa (model-perpendicular), than in layer 1,  $\sigma_x = 41.7$  MPa (model-parallel). This is somewhat anticipated considering that under bending load, the highest stress is distributed in the outer layer, which then decreases towards the neutral axis. However, the higher stress in layer 2 (model – parallel) is the consequence of a lower thickness ratio of the parallelly oriented plywood layers which are also the main bearers of stress in the plywood. When the load direction is parallel to the face grain direction, then the total number of parallelly oriented layers is six (layers 1, 3, 5, 7, 9 and 11), and when the load direction is perpendicular to the face grain direction, then the total number of parallelly oriented layers is five (layers 2, 4, 6, 8 and 10). The fact that very high stress is present in layer 2 is a very important data in

designing the structural construction of plywood.

Though markedly less than normal stress, shear stresses are also important due to the low modulus of rigidity of the wood and with that substantial strain. In shear stresses, the stress  $\tau_{xz}$  dominates, and the continued linear increase in stress from the first layer through to the 11th layer is evident. Such a linear stress distribution suggests the fact that the cross-orientation of layers does not cause stepwise values of shear stress. The remaining two shear stress components  $\tau_{xy}$  and  $\tau_{yz}$  are of equal value and substantially lower magnitude.

It is important to compare the components of shear stress with the components of strain. Namely, though the lowest stress is seen in component  $\tau_{yz}$ , this definitely causes the largest strain – component  $\gamma_{yz}$  (Tab. 2). Contrary to this, the highest shear stress is seen in component  $\tau_{xz}$ , however, this causes less strain – component  $\gamma_{xz}$ . This is the result of varying modulus of rigidity with respect to the direction of the wood. Despite the fact that stress  $\tau_{xz}$  is high, the module of rigidity of the wood  $G_{xz}$  is exceptionally high (1180 MPa) and therefore the strain is small. Unlike this, the module of rigidity of wood  $G_{yz}$  is exceptionally low (190 MPa) and therefore low stress  $\tau_{yz}$  causes the highest strain. The magnitude of the stress and strain in the model-parallel and model-perpendicular is subjected to similar regularity, with the difference that the values are somewhat larger in the model-parallel due to its lower stiffness.

The stress components in the transversal direction  $\sigma_z$ ,  $\tau_{yz}$  and  $\tau_{zx}$  are called interlaminar stresses. These through-thickness stresses do not appear only between the individual layers but also within each individual layer. Interlaminar stresses are caused by the mismatch in material properties of bonded adjacent materials (layers) and gradients in in-plane components of stress (Herakovich 1998). The appearance of high interlaminar stresses in plywood is the result of the orthotropic properties of wood, i.e. the large differences in strength with respect to the observed grain direction. Furthermore, substantial values of interlaminar stresses appear as the result of the static load, particularly when multi-layer plywood has free edges. High values of interlaminar stresses can cause delamination of plywood.

From the results of the Von Mises stress and the equivalent strain, it is evident that in the model-perpendicular, the highest Von Mises stress is in layer 2,  $\sigma_v = 49.0487$  MPa while the highest equivalent strain is in layer 1,  $\epsilon_{eq} = 0.00269049$ . In the model-parallel, the highest stress is in layer 1,  $\sigma_v = 40.8136$  MPa, as is the highest equivalent strain,  $\epsilon_{eq} = 0.00189255$ . Considering that the component of normal stress  $\sigma_x$  is markedly higher than the remaining stress components, it would appear that the value of the Von Mises stress is primarily dependent on that component. Unlike stress, equivalent strain is largely dependent on the plywood stiffness, i.e. the magnitude of the deflection. Therefore, even with smaller loads, the largest strain appears in the outer layers in the model-perpendicular, as this has a lower stiffness than the model-parallel. Namely, the maximum deflection for the model-parallel totals 4.399 mm, while for the model-perpendicular this totals 3.027 mm.

The above shows that normal stress  $\sigma_x$  is the largest and is dominant. However, the remaining stress components, despite their lower values, still play a very significant role in the analysis of stress that appears in plywood, both due to the fact that they cause substantial plywood strain and due to the need to precisely locate the parts of the surface where they appear. Therefore, it could be said that the 3D analysis allows for a detailed overview of all six stress components and, in this way, allows for the optimization of those stress components that represent critical values.

## CONCLUSIONS

In this study, a 3D analysis of stress was conducted. The results were obtained using numerical models and the bending properties of plywood were empirically derived. The results of 3D analysis include the values of all six stress components ( $\sigma_x$ ,  $\sigma_y$ ,  $\sigma_z$ ,  $\tau_{xy}$ ,  $\tau_{xz}$ ,  $\tau_{yz}$ ) and all six strain components ( $\epsilon_x$ ,  $\epsilon_y$ ,  $\epsilon_z$ ,  $\gamma_{xy}$ ,  $\gamma_{xz}$ ,  $\gamma_{yz}$ ). Also, the values of the Von Mises stress, equivalent strain and maximum deflection were included.

Unlike classical analyses, 3D analysis allowed for a detailed overview of the mutual relations of individual stress components and their influence on individual strain components. The highest values were seen in normal stress, particularly in the direction parallel to the load direction - component  $\sigma_x$ . Though the stress components  $\sigma_y$  and  $\sigma_z$  were substantially lower and equal, they are also significant as the mechanical properties of the veneer are significantly lower in their directions. In monitoring the changes in values of normal stress by thickness of the plywood, it was determined that the values of  $\sigma_y$  and  $\sigma_z$  do not show large discrepancies. Unlike these, the value  $\sigma_x$  change stepwise linearly, which is the direct result of the cross-orientation of individual layers. With respect to shear stress, the highest values were seen in the component  $\tau_{xz}$  as its direction is parallel to the load direction. The component  $\tau_{xz}$  is also characterized by the highest increase in stress in the transversal direction, growing continually linearly from the bottom up. The two remaining stress components  $\tau_{xy}$  and  $\tau_{yz}$  have lower values, however, these are also very important in the analysis of plywood stress, as they cause significant plywood strain.

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