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VECTOR OPTIMISATION OF A SINGLE-COMPARTMENT FURNITURE CONSTRUCTION

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ABSTRACT

The presented study is an attempt at carrying out a multicriterial (vector) optimisation of a single-compartment furniture construction. The following three objective functions were adopted: volume minimisation, maximisation of the furniture body total stiffness and minimisation of deviations from functional dimensions as well as six decision variables of discrete nature. Moreover, a number of limiting conditions were taken into consideration which took into account the greatest number of factors affecting the quality of the designed piece of furniture. These preconditions included, primarily: strength, rigidity and stability relationships as well as technological and functional conditions. Following the performed poly-optimisation of the analysed construction using the method of genetic algorithms, a number of non-dominated solutions were obtained allowing the decision-maker to select the best construction in accordance with the adopted criteria.

KEY WORDS: optimisation, construction, furniture

INTRODUCTION

The expectation to achieve the best possible result of actions in given conditions and at specified assessment criterion occurs in all spheres of conscious human activity. Therefore, also the introduction into production of furniture constructions checked analytically and in laboratory is associated with a desire to optimise material consumption and reduce production costs and, at the same time, to maintain high quality of the product. The cost of materials is one of the elements of the technical cost of product manufacture and, in conditions of furniture production, it makes up from 60 to 80% of Technical Cost of Production. Bearing in mind this factor, it seems expedient to introduce optimisation calculations to furniture design with the aim not only to confine to minimum the occurrence of defective solutions but also to minimise material consumption at the simultaneous maintenance of the high quality of the final product.

The first to focus on the problems of rationalisation of furniture constructions in Poland was Kotaś (1957, 1958). In his studies, this author carried out a comprehensive analysis of cabinet furniture deformation. In addition, he proposed optimisation of board thickness from which the body of this type of furniture was made by their appropriate increase or decrease and, at the same time, indicated the possibility of the application of the 'sandwich' type of plastics instead of traditional, material-consuming solid boards of considerable thickness.

Similar problems were undertaken by other researchers (Bachmann and Hässler 1981, Kŭčukov 1986, Eckelman 1990, Cai and Wang 1993, Zhang and Eckelman 1993). The problem of numerical optimisation of cabinet furniture was undertaken by (Kłos 1996) who indicated possibilities of the stiffness optimisation of the entire construction using the method of random walk for three different constructions of cabinet furniture: office bookcase, bedside cabinet and a wardrobe. In the above-mentioned study, maximisation of body rigidity was adopted as the objective function. In the result of the application of the skirting board, the total rigidity increased from: 164% - for the bookcase to 256% - for the bedside cabinet. Issues associated with the optimisation of cabinet furniture were investigated in papers published by (Smardzewski and Dzięgielewski 1997) and (Dzięgielewski and Kłos 2000) in which the authors presented an OMS-1 computer program utilising the algorithm of the random walk method. The studies presented a scalar optimisation of board thickness from which the body of a piece of furniture was constructed as well as the dimensions of a cross section of a transverse strengthening rib situated in the skirting board. These calculations aimed at determining the optimal rigidity of the system but they failed to take into account strength parameters of joints.

The above-mentioned papers from the area of construction rationalisation concerned single-factorial optimisation, i.e. the selection of such decision variables from a set of acceptable solutions for which the value of the objective function (which is the optimisation criterion) reached the extreme value. Only one objective function always occurred in these problems and a specific value was its result. Such an approach to the problem is conceptually fairly simple and not very troublesome for the algorithmization of the solution but, unfortunately, it provides only some, not always sufficient information necessary to select the construction which is really the best one. Comprehensive information about properties of the object under design can only be obtained when methods of multicriterial optimisation are applied.

In the case of the multicriterial optimisation of furniture constructions paper published by (Kłos and Smardzewski 2004) and (Kłos 2005) deserve attention. In the first of them, the authors conducted the process of poly-optimisation of a multi-compartment piece of furniture assuming two objective functions: maximisation of furniture stiffness and minimisation of consumption of board materials. The same criteria were adopted in the second study but with reference to a single-compartment body. In both cases, a number of compromise solutions were obtained in the result of performed calculations and it is up to the decision-maker to select one of these solutions depending on his/her needs or experience.

Objekt of poly-optimisation and mathematical model of the construction

Due to widespread production of openwork type of furniture, the object of multicriterial optimisation was a single-compartment piece of furniture – further on referred to as coffer – of column type construction as shown in Fig. 1.

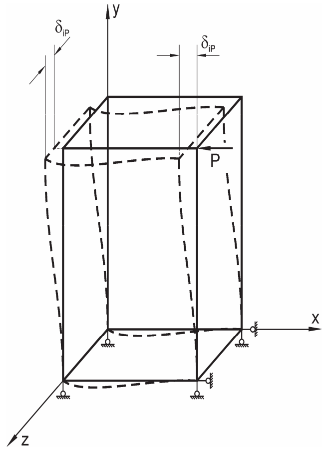

Fig. 1: Object of poly-optimisation – single compartment body
Source: own elaboration

The experimental model was manufactured from crude chipboard with wood dowels as connecting elements. The experimental model did not have its back wall fixed. In order to determine stiffness of the single-compartment body, it was necessary to know the equivalent stiffnesses of angle joints. In order to determine equivalent stiffness module of joints, Maxwell- Mohr's theory was employed whose general form (with the exception of axial and transverse forces) is presented in the equation below:

$$\delta_z = \int_x \left(\frac{M_i M_K}{EJ} \right) dx \tag{1}$$

where:
δ_z – total joint deflection
M_i – virtual moments caused by virtual load X_i
M_k – real moments caused by virtual load X_i
E –Young modulus
J – moment of inertia.

The performed calculations assumed that the board from which the joint was made consisted of two segments of different stiffness. The stiffness of the section closest to the node was $E_z J$, while the stiffness of the other section was $E_w J$.

Fig. 2 shows the basic state, diagrams of bending moments as well as the virtual state in accordance with the adopted theory for cases of compression of the connection.

Calculating the Maxwell-Mohr integer, deflexions for two different rigidities were determined:

$$\delta_1 = \frac{1}{E_w J} \int_0^{l_1} P_1 \cdot x \cdot \overline{1} \cdot x dx = \frac{P_1 l_1^3}{3 E_w J}, \tag{2}$$

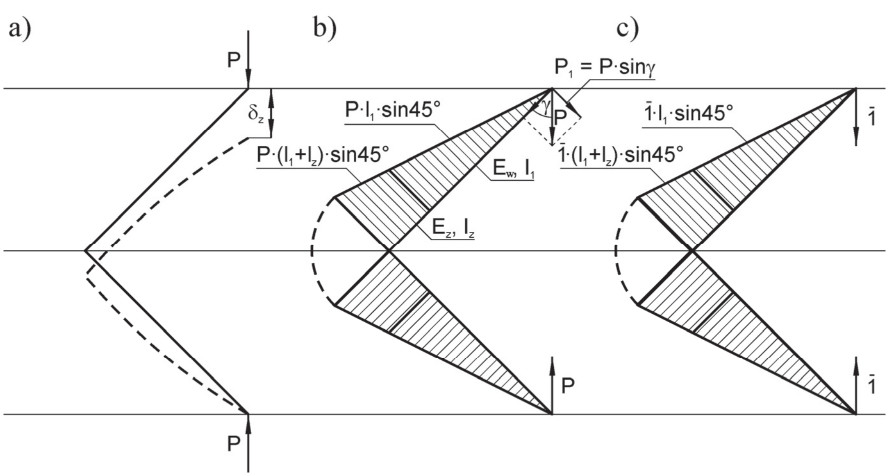

Fig. 2: Static scheme of bending of an angle joint:
a/ basic state, b/ diagram of bending moments, c/ diagram of virtual moments.
Source: own elaboration

$$\delta_2 = \frac{1}{E_z J} \int_{l_1}^{l_1+l_z} P_1 \cdot x \cdot \bar{1} \cdot x dx = \frac{P_1 l_1^3}{3E_z J}\left(3l_1^3 l_z + 3l_1 l_z^2 + l_z^3\right), \tag{3}$$

where:
δ_1 – deflexion of the arm of the joint at the length of l_1,
δ_2 – deflexion of the near-node part of the joint along the length of l_z.
$\bar{1}$ - unit virtual load
P_1 – external force.

The total deflexion of the angle joint is a sum of δ_1 and δ_2:

$$\delta_z = \delta_1 + \delta_2 = \frac{P_1 l_1^3}{3E_w J} + \frac{P_1 l_1^3}{3E_z J}\left(3l_1^3 l_z + 3l_1 l_z^2 + l_z^3\right), \tag{4}$$

where:
E_w - Young modulus of a chip board
E_z – substitute Young modulus of joint.
Since the value of the δ_z deflexion was measured in the course of laboratory experiments with the assistance of the testing machine, the equation was transformed to find E_z:

$$E_z = \frac{l_1^2 l_z + l_1 l_z^2 + \frac{1}{3} l_z^3}{\frac{\delta_z J}{P_1} - \frac{l_1^3}{3E_w}}. \tag{5}$$

The above dependence is correct for compression and stretching of the connection, but in the case of stretching, the value of the P_1 force amounts to:

$$P_1 = \frac{P}{2} \cdot \sin\gamma. \tag{6}$$

In the case of support of the construction in four corners, bending of its individual elements occurred as shown in Fig. 1. In order to determine stiffness of such system, the Maxwell-Mohr method was employed again writing it down in the form of canonical equation. The optimised coffer was a threefold internally statically indeterminable structure; therefore, a canonical equation for this construction can be presented as follows:

$$\begin{cases} \delta_{11}X_1 + \delta_{12}X_2 + \delta_{13}X_3 + \delta_{10} = 0 \\ \delta_{21}X_1 + \delta_{22}X_2 + \delta_{23}X_3 + \delta_{20} = 0 \\ \delta_{31}X_1 + \delta_{32}X_2 + \delta_{33}X_3 + \delta_{30} = 0 \end{cases} \tag{7}$$

where:

X_1, X_2, X_3 – virtual forces and moments.

Fig. 3 shows the arrangement of internal forces, disengagement state as well as diagrams of real and virtual moments.

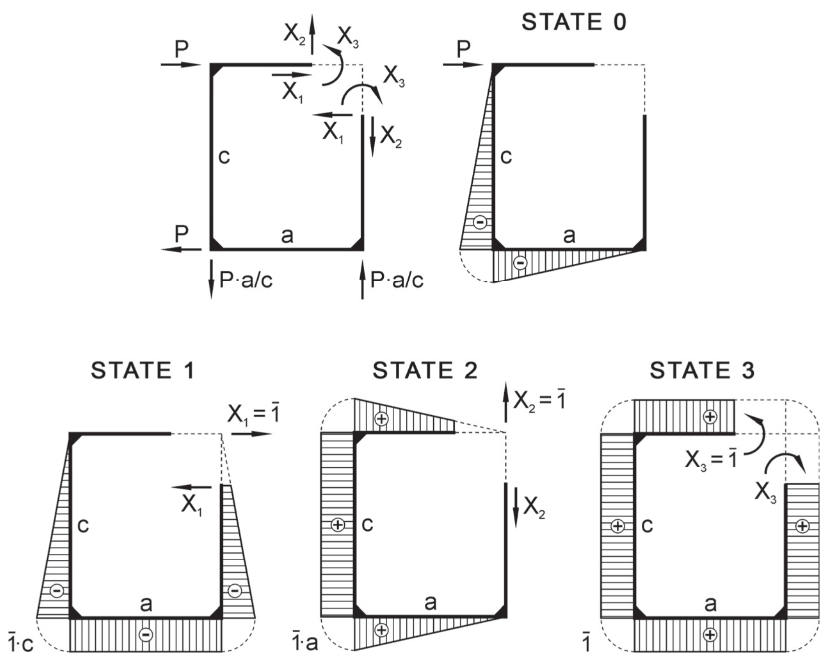

Fig. 3: Disengagement state and diagrams of real and virtual moments. STATE 0 – diagrams of real bending moments derived from force P, STATE 1 and STATE 2 - diagrams of virtual bending moments derived from forces, respectively X_1 and X_2, STATE 3 – diagrams of virtual bending moments derived from moment X_3.
Source: own elaboration

Hence, equations of internal forces assumed the following form:

$$\begin{cases} X_1 = -\dfrac{P}{2} \\ X_2 = \dfrac{Pc}{2a} \\ X_3 = -\dfrac{Pc}{4} \end{cases}$$

In order to determine the size of the body deformation under the influence of an external force, Castigliano's theorem was used, whose general form can be written down as follows:

$$u_i = \delta_{iP} = \frac{\partial U}{\partial P_i} = \sum_{i=1}^{n} \int_0^x \frac{M(x_i)}{EJ} \frac{\partial M(x_i)}{\partial P_i} dx, \tag{8}$$

where:
U – potential energy of the body
u_i – shift of point i along the direction of the action of force P

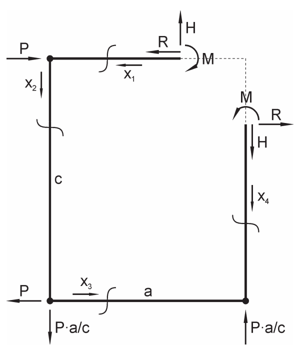

Fig. 4: Distribution of forces in the frame.
Source: own elaboration

For the considered case (without taking into consideration stiffness of joints), the above dependence can be written as follows:

$$\frac{\partial U}{\partial P} = \frac{1}{EJ}\left[\int_0^a M(x_1)\frac{\partial M(x_1)}{\partial P}dx + \int_0^c M(x_2)\frac{\partial M(x_2)}{\partial P}dx + \right.$$
$$\left. + \int_0^a M(x_3)\frac{\partial M(x_3)}{\partial P}dx + \int_0^c M(x_4)\frac{\partial M(x_4)}{\partial P}dx \right]$$

Taking into consideration signs of internal forces, the following designations were adopted:

$$X_1 = R = \frac{P}{2}; \; X_2 = H = \frac{Pc}{2a}; \; X_3 = M = \frac{Pc}{4}, \tag{9}$$

and, therefore, the distribution of forces in the frame can be presented as in Fig. 4.

The total displacement δ_{iP} was written down in the following form:

$$\delta_{iP} = \frac{Pc^2(a+c)}{24EJ} \qquad (10)$$

Taking into consideration the fact that the coffer is a board structure and making use of the basic stiffness definition of the body of a piece of furniture *k*, this was expressed by an equation:

$$k = \frac{P}{\delta_{iP}} = \frac{24D_w b\left(1-\nu^2\right)}{c^2(a+c)}, \qquad (11)$$

where:

D_w – rigidity of the board

E, ν- Young's modulus and Poisson's coefficient of the board.

The above equation assumes, in accordance with the Maxwell-Mohr theory, identical stiffness of both board elements and joints. However, such an assumption is a simplification that goes too far because, as evident from investigations (Branowski and Pohl 2004), furniture joints are semi-stiff joints. Taking this fact into consideration, the formula which describes real displacements of the upper rim in relation to the bottom coffer assumed the following form:

$$\begin{aligned}
\delta_{iP} = \frac{\partial U}{\partial P} &= \frac{1}{E_z^Z J}\int_0^{l_z} M(X_1)\frac{\partial M(X_1)}{\partial P}dx + \frac{1}{E_w J}\int_{l_z}^{a-l_z} M(X_1)\frac{\partial M(X_1)}{\partial P}dx + \\
&+ \frac{1}{E_z^R J}\int_{a-l_z}^{a} M(X_1)\frac{\partial M(X_1)}{\partial P}dx + \frac{1}{E_z^R J}\int_0^{l_z} M(X_2)\frac{\partial M(X_2)}{\partial P}dx + \\
&+ \frac{1}{E_w J}\int_{l_z}^{c-l_z} M(X_2)\frac{\partial M(X_2)}{\partial P}dx + \frac{1}{E_z^Z J}\int_{c-l_z}^{c} M(X_2)\frac{\partial M(X_2)}{\partial P}dx + \\
&+ \frac{1}{E_z^Z J}\int_0^{l_z} M(X_3)\frac{\partial M(X_3)}{\partial P}dx + \frac{1}{E_w J}\int_{l_z}^{a-l_z} M(X_3)\frac{\partial M(X_3)}{\partial P}dx + \\
&+ \frac{1}{E_z^R J}\int_{a-l_z}^{a} M(X_3)\frac{\partial M(X_3)}{\partial P}dx + \frac{1}{E_z^R J}\int_0^{l_z} M(X_4)\frac{\partial M(X_4)}{\partial P}dx + \\
&+ \frac{1}{E_w J}\int_{l_z}^{c-l_z} M(X_4)\frac{\partial M(X_4)}{\partial P}dx + \frac{1}{E_z^Z J}\int_{c-l_z}^{c} M(X_4)\frac{\partial M(X_4)}{\partial P}dx.
\end{aligned} \qquad (12)$$

Designing mathematical model

Objective function

Three objective functions were proposed for the needs of this article. The first of them assumes pursuit of material consumption minimisation as a natural criterion which, in the simplest case, can be described as proportional to the construction costs.

$$f_1(m) = \sum_{i=1}^{n} a_i b_i d_i \rho_i \Rightarrow \min \qquad (13)$$

Stiffness of the entire structure was adopted as the second objective function. This criterion belongs to the group which includes the criteria of the highest stiffness or the least deformability.

$$f_2(k) = \frac{24D_w b(1-\nu^2)}{c^2(a+c)} \Rightarrow \max \tag{14}$$

The third objective function was expressed as the ergonomic trait maximisation of the construction of a piece of cabinet furniture:

$$f_3(a,b,c) = \sqrt{\left(\frac{a-a_d}{a}\right)^2 + \left(\frac{b-b_d}{b}\right)^2 + \left(\frac{c-c_d}{c}\right)^2} \Rightarrow \min, \tag{15}$$

where:
a, b, c – dimensions of the furniture body
a_d, b_d, c_d – are deviation from the set: width, depth and height of the coffer, respectively.
ρ - board density.

Decision variables

In the performed index optimisation, the vector of decision variables assumed the following form:

$$x = (a_d, b_d, c_d, \overline{d}_1, n_k, g_k)^T ,$$

where:
a_d, b_d, c_d – deviation from ideal furniture external dimensions: discrete variable,
$\overline{d}_1$ – thickness of sides and rims: discrete variable,
n_k – number of fastening in corners (dowels or eccentrics): discrete variable,
$g_k=g_1+g_2$ – dowel length expressed as the sum of set depths in individual elements: discrete variable.

Limiting conditions

The conditions limiting the acceptable area were divided as follows:
- **Stiffness conditions** – taking into consideration global stiffness and local construction,
- **Stability conditions** – ensuring stability of a piece of furniture as well as local stability of side walls,
- **Strength conditions** – ensuring maintenance of maximum strains in joints of construction nodes below acceptable values,
- **Technological conditions** – taking into account those factors which are beyond the control of the furniture designer resulting from production limitations of raw material suppliers, manufacturing etc.,
- **Functional conditions** – enforcing limits on functional dimensions of cabinet furniture in accordance with the PN-91/F-06027/02 standard.

Method and optimisation algorithm

The general block diagram of genetic algorithms employed to implement the adopted task of multicriterial optimisation is presented in Fig. 5. The first step in the process of solving the optimisation problem using the method of genetic algorithms is the appropriate writing down of the task. The problem is coded in the form of bit strings which create a code sequence called a chromosome. The action of genetic algorithms begins with the generation of initial, random population (set of possible solutions). A certain number of the best adopted individuals is selected for each consecutive generation. Later on, these individuals are "randomly" mated into pairs (parents). Next, employing the crossing mechanism, new code sequences are obtained corresponding to a genotype. With slight probability, due to the mechanism of mutation, random changes in genes appear. The newly developed sequences represent a successive population. The process of developing next populations is terminated if the best algorithm fulfils the assumed criteria.

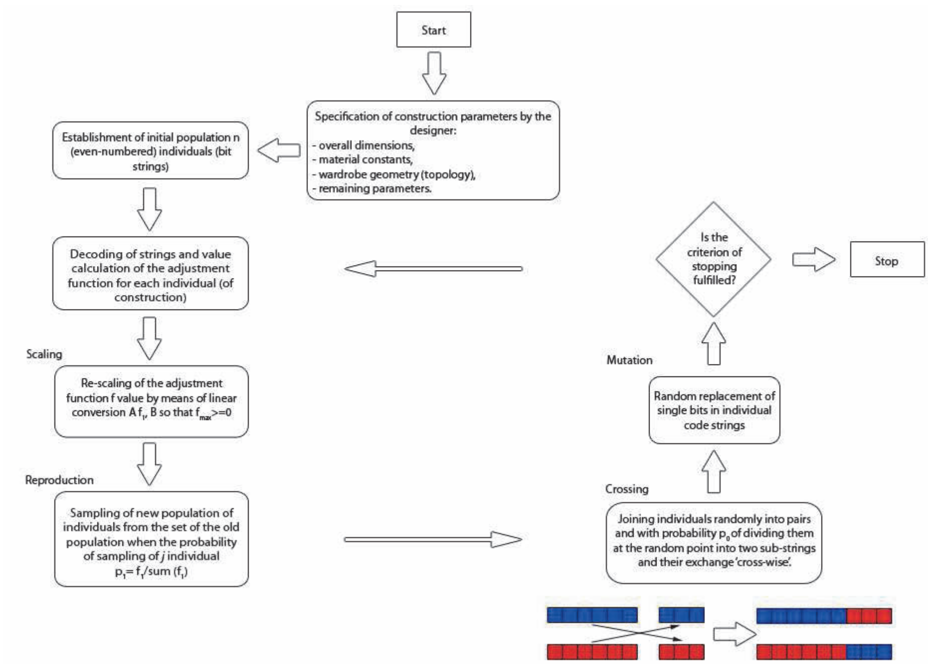

Fig. 5: Algorithms of the optimisation method Source: own elaboration

RESULTS AND DISCUSSION

In the course of the performed coffer numerical poly-optimisation with the assistance of the computer application, a number of non-dominated solutions were obtained and they are presented in Fig. 6.

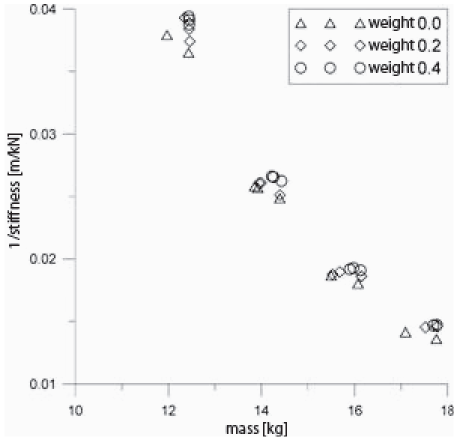

Fig. 6: Non-dominated solutions for a single-compartment furniture body.
Source: own elaboration

Taking into consideration two objective functions: stiffness maximisation and volume minimisation, the above diagram presents non-dominated solutions for a coffer. This solution was presented for the selected values of the third weight of the objective function, i.e. maximisation of ergonomic traits of a piece of furniture.

The enclosed legend includes three selected weights of the ergonomic criterion. The third objective function is not taken into consideration for the 0.0 weight, therefore the weights of the two remaining functions amount to 0.5 each. In the case of 0.2 and 0.4 weights of the ergonomic criterion, the weights of the mass and rigidity criteria amount to 0.4 and 0.4 and 0.3 and 0.3, respectively.

The grey colour in Tab. 1 indicates the adopted optimal construction. Weight values in the first row of the table designate the mass criterion weight. The stiffness criterion weight constitutes supplementation to 0.8 of the mass criterion weight.

Analysing the results of the coffer poly-optimisation in Tab. 1, it can be concluded that the greatest changes in the objective function values occurred for the mass and stiffness criteria. This situation should have been expected because these functions are antagonistic towards each other (the mass optimisation function was minimised while the rigidity optimisation function was maximised).

In the case of limiting conditions, true coffer stiffness taking into account real equivalent module of joints turns out to be the most important of them. Their low value, in comparison with stiffness criterial values assuming the same stiffness of both boards and joints, makes it necessary to accept values lower than the standard ones (10 000 $N.m^{-1}$). Therefore, a coffer whose true stiffness was higher than 3 000 $N.m^{-1}$ was accepted as a good construction.

On the other hand, the weakest condition which exerted very little impact on the optimisation process was the side wall stability condition. The value of the calculated critical force compressing the side of the body amounted to 12 500 N. However, it seems improbable for forces even similar to the calculated value to occur in normal conditions of utilisation.

Tab. 1: Poly-optimisation results of a multi-compartment furniture body

weight	0,00	0,08	0,16	0,24	0,32	**0,40**	0,48	0,56	0,64	0,72	0,80
depth [mm]	421	417	425	414	412	**427**	414	413	426	423	418
width [mm]	399	397	396	395	393	**394**	394	394	395	397	399
height [mm]	798	794	791	789	785	**787**	788	787	789	794	797
board thickness [mm]	28	28	25	22	19	**19**	19	18	18	18	18
mass [kg]	**17,78**	**17,52**	**16,14**	**15,69**	**15,53**	**14,4**	**14**	**13,95**	**12,46**	**12,44**	**12,35**
stiffness [$kN.m^{-1}$]	**68,49**	**68,86**	**53,77**	**52,78**	**53,33**	**39,82**	**38,31**	**38,35**	**26,73**	**26,06**	**25,45**
Relative deviation from dimensions [%]	**0,25**	**0,75**	**1,19**	**1,43**	**1,9**	**1,75**	**1,5**	**1,67**	**1,43**	**0,75**	**0,48**
Critical force leading to stability loss [N]	49,84	49,93	46	43,74	43,33	**41,42**	39,03	38,86	35,63	35,13	34,35
Critical force of side wall [N]	33711	33728	26246	25697	25834	**19304**	18622	18624	13016	12762	12517
True stiffness [$kN.m^{-1}$]	9,59	9,61	6,37	6,24	3,94	**3,30**	3,22	3,18	3,15	3,11	3,08
Relative rim deflexion [mm]	0,02	0,02	0,03	0,03	0,03	**0,04**	0,04	0,04	0,05	0,05	0,06
Number of dowels	2	2	2	2	3	**3**	3	4	4	5	5

CONCLUSIONS

1. Multicriterial numerical optimisation of cabinet furniture allows finding constructions which are superior with regard to the adopted stiffness and mass criteria as well as ergonomic properties.
2. When planning further investigations, it appears necessary to carry out optimisation of furniture production technology in real production conditions. A synthesis of construction, technological and aesthetic criteria would make it possible to introduce into the market of full-quality products of low production costs.

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