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DIRECTIONS DYNAMIC MODULI OF ELASTICITY WITH TRANSFORMATIONS INTO ANATOMICAL

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ABSTRACT

This paper points out that it is necessary to transform values measured in geometrical directions into anatomical directions before evaluating and comparing them with the values from the literature. This is common for practical situations, where it is impossible to determine the properties in clear anatomical directions.

KEY WORDS: dynamic modulus of elasticity, anatomical directions, geometrical directions, engineering constant in wood, transformations.

INTRODUCTION

The most important condition for the determination and evaluation of the actual state of a historical object is the non-destructivity of the used diagnostic methods for the sake of minimum intervention into historical construction. This condition can be fulfilled by using the Arborsonic Decay Detector diagnostic device. This can disclose hidden cavities and dry rot. The principle of the operation lies in the measurement of ultrasound velocity through the block. In this way it is possible to determine the dynamic modulus of elasticity by means of the known relations as one of the characteristics of the material. From the practical viewpoint, it is not possible to measure the data in the ideal radial or tangential directions, but only in the geometrical direction. This paper is concerned with the inevitable transformation of values from geometrical to anatomical directions. In a 2D situation, the relations between engineering constants and transformed engineering constants are documented in (Bodig and Jayne 1982), (Požgaj et al. 1997), (Liu and Ross 1998), citing (Jones 1975, Daniel and Ishai 1994).

Ultrasound testing

Ultrasound testing is based on the measurement of the propagation velocity of the elastic strain and it depends on the direction of the signal transmission. This signal is represented by ultrasound waves. In elastic materials, there are elements that are mutually bound or affected in a mutual crash. These binds cause the oscillation of elements to be transferred in continuum and thus the mechanical wave's motion occurs, which is accompanied by a rise in elastic waves. The ultrasound waves are propagated with velocity depending on the material and the surrounding conditions such as pressure, temperature and the relative atmospheric humidity and moisture content of the material. Their intensity is decreased with propagation on a bigger wave front and this is due to partial absorption in the material and reflection from elements. The mechanical properties of anisotropic and orthotropic materials are known to be strongly dependent on the orientation of the reference coordinate system. These properties are related by certain tensor transformation rules when the reference coordinate system changes from one orientation to another. They depend on changes in the relative prolongation of a short section in the elongated rod through longitudinal oscillation. The velocity of mechanical wave propagation can be derived from differential equations (Bucur 1995). The relative prolongation is a normal deformation of elements, as is known from the tensor of small deformations, for example:

$$\varepsilon_{ii} = \frac{\Delta u_i}{\Delta x_i} \qquad (1)$$

Where ε is relative prolongation, *u* – reformation and *x* – the original proportion of the rod section

It is possible to prove that the common wave equation is defined as:

$$\frac{\partial^2 u_1}{\partial x^2_1} - \frac{1}{c^2}\frac{\partial^2 u_1}{\partial t^2} = 0 \qquad (2)$$

Where *c* is the phase velocity of propagation of plane harmonic waves along axis x_1

From this equation, it follows that the longitudinal mechanical wave's motions are expanding with velocity along the rod:

$$c_1 = \sqrt{\frac{C_{11}}{\rho}} \qquad (3)$$

$$C_{11} = \frac{E_{11}}{1 - \mu_{12}\mu_{21}} \qquad (4)$$

μ_{12} - Poisson's ratio

The velocity of propagation of ultrasound waves in wood can be determined from the relation:

$$c = \sqrt{\frac{E}{\rho}} \qquad (5)$$

where *E* is the Young modulus of elasticity (Pa), ρ is the density of the wood ($kg.m^{-3}$)

Consequently, it is possible to calculate the dynamic modulus of elasticity, from the measured value of the velocity and density (Požgaj et al. 1993)

$$E = c^2\rho \qquad (6)$$

As follows from equation (6), the higher the velocity of the ultrasound propagation, the higher is the modulus of elasticity and the lower is the density of the wood. The velocity of propagation of ultrasound depends on the wood type and it is also of an anisotropic nature. The time of stress wave propagation in wood is influenced by imperfections in the wood, by the presence of cavities, inorganic elements or by wood decay. If the block of wood is degraded by biotic factors, a significant prolongation of the time of ultrasound wave propagation occurs.

Transformation relations for rngineering constants on a 2D plane

Wood may be described as an orthotropic material with independent mechanical properties in the directions of three mutually perpendicular axes – longitudinal (*L*), radial (*R*), and tangential (*T*). These are called the principal material axes, and the related mechanical properties are the engineering constants. The material axes and the geometrical axes used to describe a rectangular structural member do not usually coincide. The mechanical properties referring to the geometrical axes are called the transformed engineering constants. The 22–33 coordinate systems are shown in Fig. 1, which represents the principal material axes. The *x–y,* coordinate system represents the geometrical axes. $\bar{E}_i$ are geometrical axes and E_i are anatomical axes.

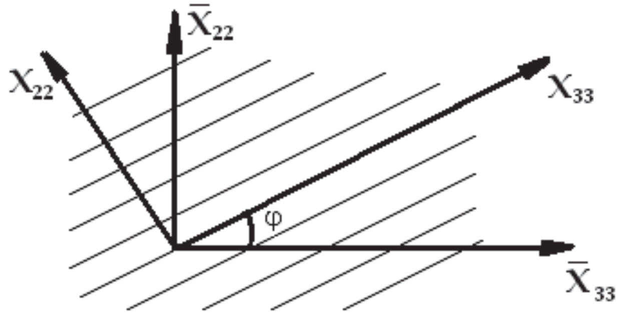

Fig. 1: Anatomical and geometrical axes in wood

Transformation of stress and strain

2 coordinate systems: $\bar{x}_i$ **geometrical axes**

x_i **anatomical axes**

From Hook's law:

$$\varepsilon = S\sigma \tag{7}$$

$$\bar{\varepsilon} = \bar{S}\bar{\sigma} \tag{8}$$

$$\bar{\sigma} = T_\sigma.\sigma \tag{9}$$

$$\bar{\varepsilon} = T_\varepsilon.\varepsilon \tag{10}$$

Transformation matrix (Bodig and Jayne 1982):

$$T = \begin{bmatrix} \cos^2\varphi & \sin^2\varphi & 2\sin\varphi\cos\varphi \\ \sin^2\varphi & \cos^2\varphi & -2\sin\varphi\cos\varphi \\ -\sin\varphi\cos\varphi & \sin\varphi\cos\varphi & \cos^2\varphi - \sin^2\varphi \end{bmatrix} \tag{11}$$

$$\overline{S} = T_\varepsilon S T_\sigma^{-1} \tag{12}$$

$$S = T_\varepsilon^{-1} \overline{S} T_\sigma \tag{13}$$

An inverse matrix can be obtained:

$$T^{=1} = \begin{bmatrix} \cos^2\varphi & \sin^2\varphi & -2\sin\varphi\cos\varphi \\ \sin^2\varphi & \cos^2\varphi & 2\sin\varphi\cos\varphi \\ \sin\varphi\cos\varphi & -\sin\theta\cos\varphi & \cos^2\varphi - \sin^2\varphi \end{bmatrix} \tag{14}$$

And this relationship indicates:

$$S_{22} = \overline{S}_{22}\cos^4\varphi + S_{33}\sin^4\varphi + 2(\overline{S}_{23} + S_{44})\cos^2\varphi\sin^2\varphi \tag{15}$$

$$S_{33} = \overline{S}_{33}\cos^4\varphi + S_{22}\sin^4\varphi + 2(\overline{S}_{23} + S_{44})\cos^2\varphi\sin^2\varphi \tag{16}$$

if $\sigma = E\varepsilon$ then $$S_1 = \frac{1}{E_1} \tag{17}$$

To obtain the geometrical values of properties:

$$\overline{E}_{22} = \frac{E_{22}E_{33}}{E_{22}\cos^4\varphi + E_{33}\sin^4\varphi + \left(\frac{1}{G_{23}} - 2\frac{\mu_{23}}{E_{22}}\right)E_{22}E_{33}\cos^2\varphi\sin^2\varphi} \tag{18}$$

$$\overline{E}_{33} = \frac{E_{22}E_{33}}{E_{33}\cos^4\varphi + E_{22}\sin^4\varphi + \left(\frac{1}{G_{23}} - 2\frac{\mu_{23}}{E_{22}}\right)E_{22}E_{33}\cos^2\varphi\sin^2\varphi} \tag{19}$$

Lui and Ross (1998), citing Jones (1975), verified this relation (28) and they obtained numerical results for the transformation of modulus of elasticity and shear modulus engineering constant in spruce *Picea sitchensis*. But for practical reasons, for example in civil buildings' investigations, opposite processes are needed.

To obtain the anatomical values of properties from a geometrical direction:

$$E_{22} = \frac{\overline{E}_{22}\overline{E}_{33}}{\overline{E}_{22}\cos^4\varphi + \overline{E}_{33}\sin^4\varphi + \left(\frac{1}{\overline{G}_{23}} - 2\frac{\overline{\mu}_{23}}{\overline{E}_{22}}\right)\overline{E}_{22}\overline{E}_{33}\cos^2\varphi\sin^2\varphi} \tag{20}$$

$$E_{33} = \frac{E_{22}E_{33}}{\overline{E}_{33}\cos^4\varphi + \overline{E}_{22}\sin^4\varphi + \left(\frac{1}{\overline{G}_{23}} - 2\frac{\overline{\mu}_{23}}{\overline{E}_{22}}\right)\overline{E}_{22}\overline{E}_{33}\cos^2\varphi\sin^2\varphi} \tag{21}$$

MATERIAL AND METHODS

Measurements were carried out on 55 clear spruce specimens [(*Picea abies* L. (Karst.)]. They had the following dimensions: a cross-section of 50x50 mm and a length of 100 mm. These blocks were chosen very carefully so that they did not include any wood defects. Specimens were taken from built-in blocks of four different historical buildings of a different kind, age and construction and from one new beam, not yet built-in:

1. Spruce overhead beams from the agricultural wing of Pernštejn Castle (1460/61)
2. Spruce overhead beams from a burgher house in Barborská Street No. 34, Kutná Hora (1810/11)
3. Spruce overhead beams from a burgher house in Havelská Street No. 15, Praha (1809/10)
4. Spruce beams from the vicarage roof in Náměšť nad Oslavou (1854/55)
5. Not yet built-in, new spruce beam

Fig. 2: Specimen's angle of annual rings and edges

When obtaining specimens from real historical buildings, it was not possible to select sufficient reference specimens with ideal radial and tangential directions. The time of ultrasound wave propagation was measured by means of the Arborsonic Decay Detector diagnostic apparatus, with a frequency of 77 kHz between transducers and a diameter of 25 mm, which are attached opposite in two cross-sectional directions. The ultrasound velocity was calculated from this time. Tests were supplied by using standard tests of the physical properties of wood according to ČSN EN standards class 49, namely the measurement of density (ČSN EN 49 0108) and moisture content (ČSN EN 49 0103). The dynamic modulus of elasticity in both directions was determined by using relation (6).

The values of dynamic modulus of elasticity are the function of

$$E = \left(\overline{E}_{22}, \overline{E}_{33}, \overline{S}_{44}, \overline{S}_{23}\right) \tag{22}$$

Where $\overline{E}_{22}, \overline{E}_{33}$ are dynamic moduli of elasticity measured in geometrical radial and tangential directions. Values $\overline{S}_{44}, \overline{S}_{23}$ are determined based on the engineering constant for the wood type; see Tab. 1.

$$\overline{S}_{44} = \frac{1}{\overline{G}_{23}} = S_{44}\cos^2 + S_{55}\sin^2; S_{44} = \frac{1}{G_{23}}; S_{55} = \frac{1}{G_{13}} \tag{23}$$

$$\overline{S}_{23} = -\frac{\overline{\mu}_{23}}{E_{22}}; \overline{\mu}_{23} = \mu_{23}\cos^2 + \mu_{13}\sin^2 \qquad (24)$$

Tab. 1: Engineering constants used for calculating elastic components (Wood Handbook 1999)

Constants	μ_{23}	μ_{13}	G_{23} (MPa)	G_{13} (MPa)
	0.435	0.467	297	603.9

It is necessary to consider mainly the cross-sectional directions, i.e. the tangential and radial, when diagnosing the actual state of built-in wood blocks. Measured values were transformed by relations (20) and (21). For this reason, the deviation of the annual rings from the edges of specimens was determined.

RESULTS AND DISCUSSION

Transverse wave investigation was applied by means of testing using ultrasonic transducers. Ultrasound travels at different speeds through different materials and is affected by many conditions. The velocity of ultrasound propagation was measured with the Arborsonic Decay Detector defectoscopic apparatus using sound spruce wood specimens. The dynamic modulus of elasticity was determined by means of relation (6). Values were calculated on values with moisture w = 12 %. The relationship between the untransformed modulus of elasticity and the angle of annual rings is shown in Fig. 3.

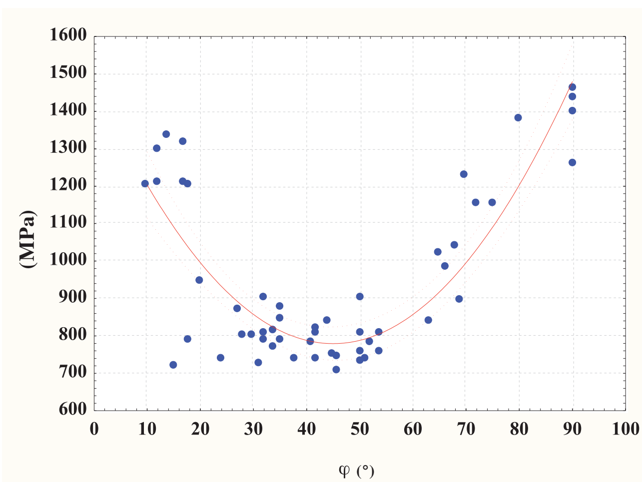

Fig. 3: Relationship between the modulus of elasticity in geometrical direction and the angle of annual rings, where 0° is the geometrical radial direction and 90° is the geometrical tangential direction

There is evident polynomial relation between these characteristics with a coefficient of correlation R^2 = 0.50. The statistically significant influence of the annual rings angle on the value of modulus of elasticity, p = 0.00, has been proved. The statistical significance of the nonlinear model has also been confirmed. The shape of the graph is close to the results published by Kloiber and Kotlínová (2004). Pellerin and Ross (2002) show close results for velocity of ultrasound wave's propagation, however, according to (6), the dynamic of the elasticity will have the same curve. Thus, results are comparable and it is evident that the dynamic modulus of elasticity is the lowest at 45° of the annual rings angle. This could be caused by the anatomical orientation of wood cells. Thus, it is clear that transformation is necessary.

Values of modulus of elasticity were transformed with the help of transformed matrixes into anatomical directions, thus into radial and tangential directions. Note that E_{33} is the dynamic modulus of elasticity in the anatomical tangential direction, E_{22} is the dynamic modulus of elasticity in the anatomical radial direction and $\bar{E}_{22}$ and $\bar{E}_{33}$ are values in the geometrical axis. Transformed values of the dynamic modulus of elasticity and untransformed values are statistically compared in Tab. 2. The average values of selected files have been tested by means of Student's t-test and variances of these files were compared by means of the Fisher F-test.

Tab. 2: Statistical evaluation of modulus of elasticity in geometrical and anatomical directions

	Mean	**Std.Dv.**	**Coefficient of variability**	**P**
E_{22}	1125.8	230.1824	0.20	0,00
E_{33}	722.3	104.4708	0.14	
$\bar{E}_{22}$	1022.2	304.0800	0.30	0,04
$\bar{E}_{33}$	958.8	278.0761	0.30	

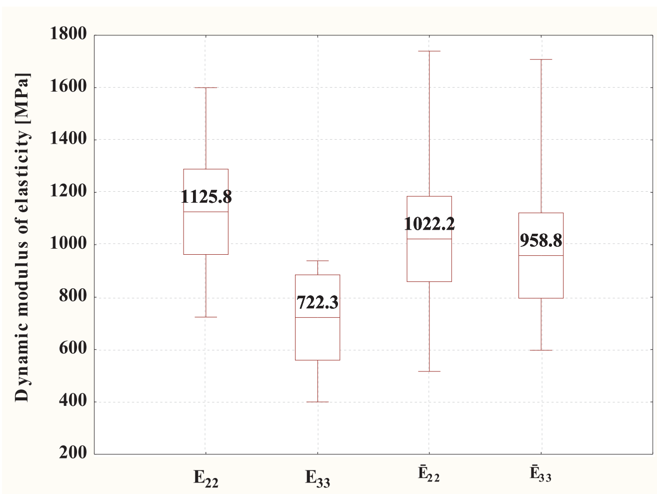

Fig. 4: Values of the dynamic modulus of elasticity for geometrical directions $\bar{E}_{22}$ $\bar{E}_{33}$ and anatomical directions E_{22} and E_{33}

Values, measured in cross-sectional geometrical directions, are not very different as is evident from Fig. 4 and statistically verified in Tab. 2 in statistical significance at α=0.05. Consequently, it is necessary to transform them by means of the transformation equations into anatomical directions. Then they can be compared with the values from literature. It was proved by the measurement, that the value of the dynamic modulus of elasticity is lower in the tangential direction, in comparison with values in the radial direction. The Wood Handbook (1999) states that the ratio between the modulus of elasticity in the tangential direction to the modulus of elasticity in the radial direction is in the following relationship:

$$\frac{E_T}{E_R} = \frac{0.043}{0.078} = 0.55 \tag{25}$$

where E_T to E_R is the ratio between static moduli of elasticity in tangential and radial directions.

Ratio of transformed values

$$\frac{E_{33}}{E_{22}} = 0.64 \tag{26}$$

The ratio between determined moduli of elasticity is very close to values from references (Wood Handbook, 1999). Differences between values in the radial and tangential directions can be related to the structure of wood and to the organization of wood cells in these directions. Values of the dynamic modulus of elasticity in both directions are higher than those of the static modulus of elasticity. There is a linear dependence, which is described by several authors, e.g. Wang et al. (2003), Booker et al. (2003), Ross et al. (1991), Oliviera (2003); but, the exact relationship was not determined. All the authors found their relations with different coefficients for describing correlations between static and dynamic moduli of elasticity.

CONCLUSION

This article presents evidence of the necessity of transformation. It is evident from the graph that untransformed values provide an error of calculation of the modulus of elasticity. The modulus of elasticity values were measured in two cross-sectional directions using sound spruce specimens by means of the defectoscopic Arborsonic Decay Detector. It has been proved that it is necessary to transform values of the dynamic modulus of elasticity measured in built-in wood from the geometrical direction to anatomical directions because it is not possible to keep precise radial and tangential directions, which is the reason why the results are distorted.

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