

## **DETERMINATION OF ELASTIC CONSTANTS OF PARTICLEBOARD LAYERS BY COMPRESSING GLUED LAYER SPECIMENS**

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### **ABSTRACT**

The method for obtaining elastic constants of the particleboard layers by compressing the block specimens glued from strips of layers separated from board, and measuring specimen deformations by the electric resistance strain gauge technique was presented. The layers were assumed to be orthotropic materials and the set of elastic constants: three Young's moduli, six Poisson's ratios and three shear moduli were determined for both the face and core layers. The effect of the glue lines contained in the specimens on the results of calculating the layer elastic constants was regarded.

**KEYWORDS:** Particleboard, face layer, core layer, Young's modulus, Poisson's ratio, shear modulus.

### **INTRODUCTION**

The A typical commercial particleboard is a three-layer panel which consists of a central layer (core) and two outer layers (faces). Structures and properties of these layers differ significantly. Knowledge of mechanical properties of these layers is required for rational designing structural members made of particleboards and for analyzing stresses and deformations in these members. Besides, it may allow to apply the theory of layered systems to particleboards (Keylwerth 1958, Hänsel et al. 1988, Bodig and Jayne 1993, Suo and Bowyer 1995, Wong et al. 2003, Wilczyński and Kociszewski 2007). The data on the layer mechanical properties may be also useful for optimization of the particleboard structure (Kühne and Niemz 1980).

Determination of mechanical properties, including elastic properties of particleboard layers is complex and labour-consuming. Probably therefore few studies have been carried out in relation to these layer properties. Selected elastic constants were determined using different method. Keylwerth (1958) studied the Young's moduli in tension and compression parallel to the layer plane for the face and core layers separated from the board, and the modulus of elasticity in bending and shear modulus in the layer plane but only for the core layer. Geimer et al. (1975) determined the Young's moduli of the face and core layers separated from the board using a tensile test in the direction

parallel to the layer plane, and the shear modulus of the core layer in the plane perpendicular to the layer by an interlaminar shear test. Dueholm (1976) studied the Young's moduli of the particleboard layers determining the face layer modulus by an indirect method consisting in comparing the stiffness of the raw and sanded specimens. He used a static tensile test and two dynamic tests. The Young's moduli of the layers were also investigated by May (1983), who determined the face layer modulus in a similar indirect way by employing a static bending test. Bucur et al. (1998) determined the Young's moduli of the face and core layers in the direction parallel to the layer plane using the ultrasonic method and dynamic mechanical analysis technique. Wilczyński and Kociszewski (2007) studied the layer moduli of elasticity in bending for three directions: the mat forming direction, the direction perpendicular to it, and the direction perpendicular to the layer plane, using the specimens glued from layers separated from boards.

Another way of evaluating the elastic properties of particleboard layers was employed by Wong et al. (2003). They assumed that particleboard consists of plies of homo-profiles boards with various densities and determined elastic properties of such boards laboratory fabricated. Three elastic parameters were determined: the Young's modulus parallel to the board plane by a static bending test, and the Young's modulus perpendicular to the board plane and the shear modulus in the board plane by dynamic methods.

In our previous study (Wilczyński and Kociszewski 2003) elastic properties of the particleboard layers were assessed, assuming that the layers are isotropic in their planes. Elastic constants for the face and core layers as the transversal isotropic materials were determined using a compression test. This study is an extension of the previous one. Orthotropic anisotropy is assumed for the layers of three-layer particleboard. Because the layer specimens were glued from plies, the effect of glue lines was considered. Determining the full set of the orthotropic elastic constants of the particleboard layers by one universal method was the objective of this study.

### Theoretical considerations

Three-layer particleboard (Fig. 1) can be regarded as an orthotropic body (Bodig and Jayne 1993, Bucur et al. 1998). The principal axes of elasticity have the following directions:  $x$  – mat forming direction,  $y$  – perpendicular to the mat forming direction, and  $z$  – perpendicular to the board.

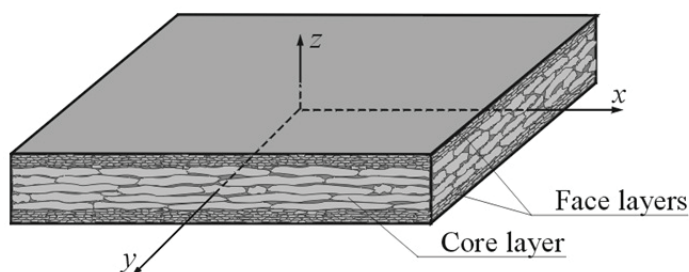


Fig. 1: Principal axes of the elasticity of three-layer particleboard

Hooke's law for the particleboard as an orthotropic material can most conveniently be written as the compliance matrix equation where strains are stated as linear functions of stresses:

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & 0 & 0 & 0 \\ S_{21} & S_{22} & S_{23} & 0 & 0 & 0 \\ S_{31} & S_{32} & S_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & S_{66} \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{bmatrix} \quad (1)$$

Substituting the compliance coefficients  $S_{ij}$  by the engineering elastic constants, Hooke's law can be expressed in the following form:

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \frac{1}{E_x} & -\frac{\nu_{yx}}{E_y} & -\frac{\nu_{zx}}{E_z} & 0 & 0 & 0 \\ -\frac{\nu_{xy}}{E_x} & \frac{1}{E_y} & -\frac{\nu_{zy}}{E_z} & 0 & 0 & 0 \\ -\frac{\nu_{xz}}{E_x} & -\frac{\nu_{yz}}{E_y} & \frac{1}{E_z} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{yz}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{xz}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{xy}} \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{bmatrix} \quad (2)$$

where:  $E_x$ ,  $E_y$  and  $E_z$  are the Young's moduli,  $\nu_{xy}$ ,  $\nu_{yx}$ ,  $\nu_{yz}$ ,  $\nu_{zy}$ ,  $\nu_{zx}$  and  $\nu_{xz}$  are the Poisson's ratios, and  $G_{yz}$ ,  $G_{xz}$  and  $G_{xy}$  are the shear moduli.

The compliance matrix is symmetric,  $S_{ij} = S_{ji}$ . Hence, one obtains the following relationships:

$$\frac{\nu_{xy}}{E_x} = \frac{\nu_{yx}}{E_y}, \quad \frac{\nu_{xz}}{E_x} = \frac{\nu_{zx}}{E_z}, \quad \frac{\nu_{yz}}{E_y} = \frac{\nu_{zy}}{E_z} \quad (3)$$

The particleboard layers, both face and core ones, can be also regarded as orthotropic materials, and the presented Hooke's law concerns these layers.

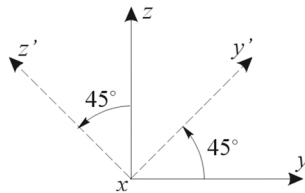


Fig. 2: Rotation of orthotropic axes in the yz plane

Determination of the particle layer Young's moduli  $E_x$  and  $E_y$  is relatively simply. They can be obtained by direct ways (Keylwerth 1958, Geimer et al. 1975, Wilczyński and Kociszewski 2007)

or indirect methods (Dueholm 1976, May 1983). Determination of the others elastic constants of the layers is more complicated. In order to determine the layer shear moduli, it is useful to consider the compliance coefficients  $S_{ij}$  in the  $x, y, z$  system, which is rotated in relation to the principal axes of elasticity. These coefficients are related to the coefficients  $S_{ij}^*$  with appropriate relations described by Hearmon (1948) and Keylwerth (1951). Particularly for the system of axes rotated by an angle of  $45^\circ$  about the  $x$  axis, see Fig. 2, there is a simple relationship (Keylwerth 1951):

$$2(S_{22}^* - S_{23}^*) = S_{44} \quad (4)$$

where:  $S_{22}^*$  and  $S_{23}^*$  are the compliance coefficients related to the system of axes rotated by an angle of  $45^\circ$  around the  $x$  axis, and  $S_{44}$  is the compliance coefficient related to the principle  $x, y, z$  axes of elasticity.

Considering the following relationships between compliance coefficients and engineering elastic parameters:

$$S_{22}^* = \frac{1}{E_y^*}, \quad S_{23}^* = -\frac{\nu_{zy}^*}{E_z^*}, \quad S_{44} = \frac{1}{G_{yz}} \quad (5)$$

and equation:

$$\frac{\nu_{yz}}{E_y} = \frac{\nu_{zy}}{E_z} \quad (6)$$

one obtains a formula in the known form:

$$G_{yz} = \frac{E_y^*}{2(1 + \nu_{yz}^*)} \quad (7)$$

By rotating the principal axes of elasticity around the  $y$  and  $z$  axes, the shear moduli  $G_{xz}$  and  $G_{xy}$  can be derived in a similar manner. The general expression for the shear modulus  $G_{ij}$  in the  $ij$  plane is:

$$G_{ij} = \frac{E_i^*}{2(1 + \nu_{ij}^*)}, \quad i, j = x, y, z; \quad i \neq j \quad (8)$$

where:  $E_i^*$  is the Young's modulus for the direction in the  $ij$  plane, forming the angle of  $45^\circ$  to the  $x$  axis,  $\nu_{ij}^*$  is the Poisson's ratio relating passive and active strains for two mutually perpendicular directions in the  $ij$  plane and forming angles of  $45^\circ$  to the  $i$  and  $j$  axes.

Eq. (8) permits the calculation of the shear modulus  $G_{ij}$  through experimental determination of  $E_i^*$  and  $\nu_{ij}^*$  parameters.

## MATERIAL AND METHODS

Typical commercial three-layer particleboard with the thickness of 18 mm and mean density of  $697 \text{ kg}\cdot\text{m}^{-3}$ , bonded with the UF resin, meeting the requirements of EN 312, grade P4 was used in the study. Density profile of this board, determined by means of laboratory density analyser GreCon DAX5000 is presented in Fig. 3.

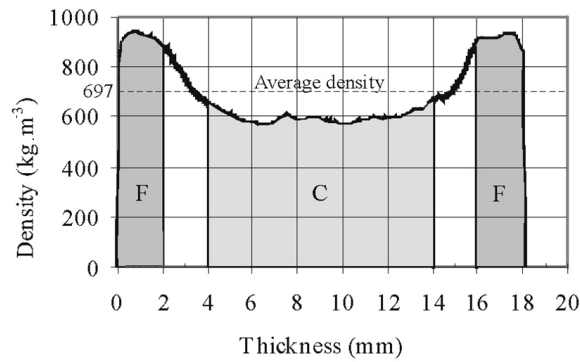


Fig. 3: Typical density profile of tested particleboard; F-face layer, C-core layer

A method based on compressing test specimens and measuring their elastic strains by means of electric resistance gauges was used for determining all elastic constants of particleboard layers. Six types of specimens 60 mm long, 30 mm wide and 20 mm thick were employed. They were glued from core or face layers separated from the board. The specimens made of core layers are presented in Fig. 4. Those made of face layers were similar but consisted of a greater number of layers. The specimens with the x, y and z longitudinal axes, shown in Figs. 4 a, b, c, were used for determining the Young's moduli and the Poisson's ratios. The diagonal specimens with the longitudinal axes forming angles of 45° to the x and y axes in the xy plane (Fig. 4d), the x and z axes in the xz plane (Fig. 4e), and y and z axes in the yz plane (Fig. 4f), were used for determining the shear moduli.

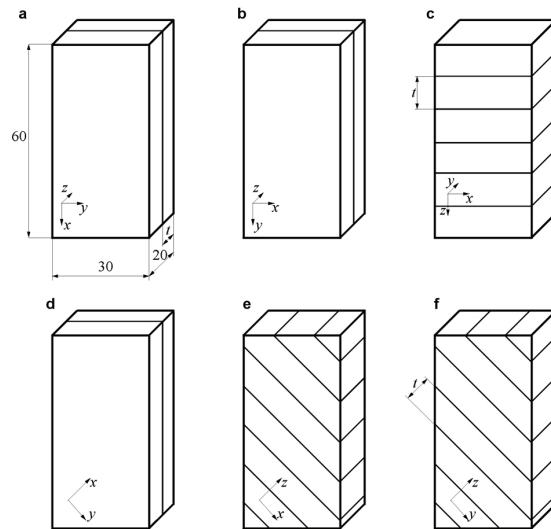


Fig. 4: Test specimens for determination of the elastic constants; a)  $E_x, \nu_{xy}, \nu_{xz}$ , b)  $E_y, \nu_{yx}, \nu_{yz}$ , c)  $E_z, \nu_{zx}, \nu_{zy}$ , d)  $G_{xy}$ , e)  $G_{xz}$ , f)  $G_{yz}$  of the particleboard core layers; t – layer thickness

### Preparation of specimens

Strips of face and core layers with the thickness of 2 and 10 mm, respectively, were separated from particleboard by appropriate mechanical processing (Fig. 5). They were glued into blocks, using the adhesive Racoll Express 25, with 160 g spread per 1 m<sup>2</sup>. The core layer blocks with 20 mm thickness were formed from two strips (Fig. 6a). The specimens presented in Figs. 4 a, b, d were cut from these blocks. Similar face layer blocks were glued from ten strips to obtain the face layer specimens. The core layer blocks 80 mm thick consisted of eight strips (Fig. 6b) and the specimens pictured in Figs. 4 c, e, f were cut from them. Similar face layer blocks consisted of forty strips and were used to prepare the face layer specimens.

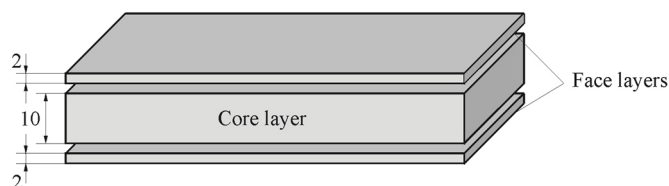


Fig. 5: Separated layers of particleboard

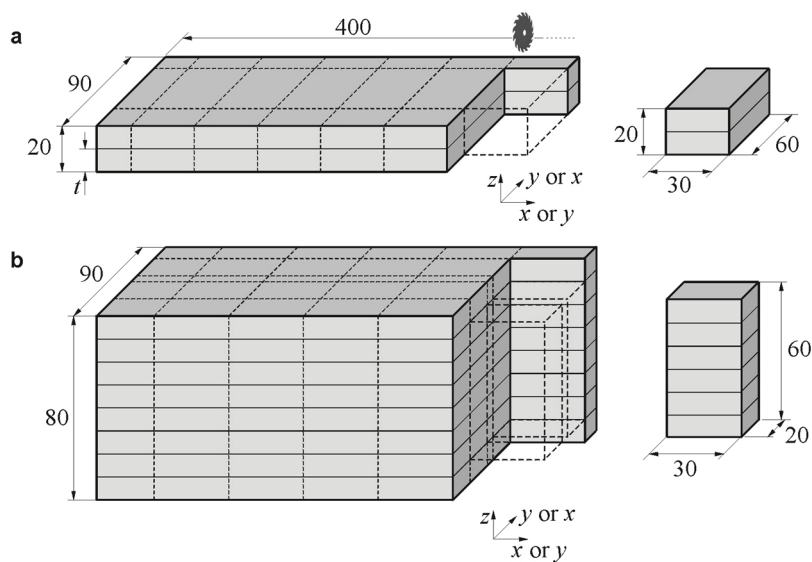


Fig. 6: Blocks of glued core layers for preparation of the specimens presented: a) in Figs 4 a,b,d and b) in Figs. 4 c,e,f

In total 60 specimens were prepared, five for each layer and type. All the specimens were conditioned to constant mass at 20°C and 65 ± 5 % relative humidity for two weeks. Density of the specimens was determined. Its mean values for face and core layers were equal 968 and 592 kg.m<sup>-3</sup>, respectively. The electric resistance strain gauges with the measurement base of 10 and 15 mm were glued onto the specimens (Figs. 7 and 8). They were placed symmetrically on the visible and opposite sides of the specimen and serially connected to the Wheatstone bridge that

allowed measuring strains with an accuracy of 1  $\mu$ S.

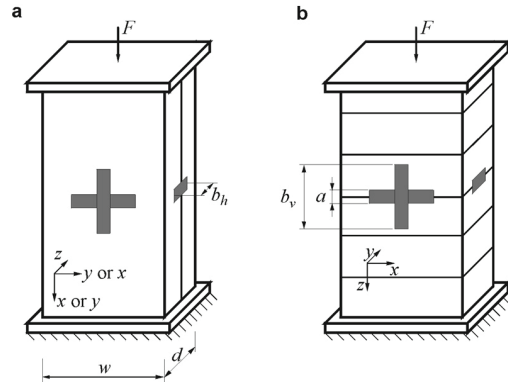


Fig. 7: Compression testing of specimens with a) the x or y and b) the z longitudinal axis

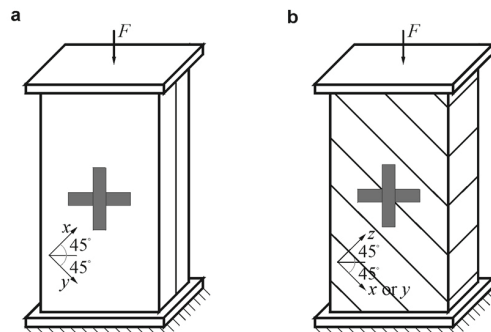


Fig. 8: Compression testing of diagonal specimens in which a) the x and y axes and b) the z and x (or y) axes form the angle of 45° to the longitudinal axis of specimen.

**Determination of the Young’s moduli  $E_x$  and  $E_y$**

The specimens are multilayer bodies consisting of board layers and glue lines. Both kinds of plies affect the specimen strains measured by electric resistance gauges. Under the action of the compressive force in the x direction (Fig. 7a) the measured strain of the specimen  $\epsilon_{xm}$ , the strain of the board layer  $\epsilon_x$ , and the strain of the glue line  $\epsilon_{xg}$  are the same:

$$\epsilon_{xm} = \epsilon_x = \epsilon_{xg} \tag{9}$$

The stresses in the board layer  $\sigma_x$  and glue line  $\sigma_{xg}$  are given by

$$\sigma_x = \epsilon_x E_x, \quad \sigma_{xg} = \epsilon_{xg} E_{xg} \tag{10}$$

where:  $E_x$  and  $E_{xg}$  are the Young’s moduli in the x direction of the board layer and glue line, respectively. The total force F on the specimen is shared by the board layers and the glue lines,

$$F = F_l + F_g \quad (11)$$

but

$$F = \sigma_{xm}wd, \quad F_l = \sigma_x w(d - d_g), \quad F_g = \sigma_{xg}wd_g \quad (12)$$

where:  $w$  and  $d$  are the width and thickness of the specimen, respectively, and  $d_g$  is the sum of the glue line thicknesses. The average stress in the entire specimen is given by

$$\sigma_{xm} = \varepsilon_{xm}E_{xm} \quad (13)$$

where:  $E_{xm}$  is the Young's modulus in the  $x$  direction of the specimen as the layered system consisting of board layers and glue lines.

Combining Eqs. (9) – (13) gives the following relation:

$$E_{xm}d = E_x(d - d_g) + E_{xg}d_g \quad (14)$$

which can be rearranged to an expression for the modulus of elasticity of the board layer, the face and core ones, in the  $x$  direction:

$$E_x = E_{xm} \frac{d}{d - d_g} - E_{xg} \frac{d_g}{d - d_g} \quad (15)$$

The modulus  $E_{xm}$  was calculated by

$$E_{xm} = \frac{\Delta F}{wd\Delta\varepsilon_{xm}} \quad (16)$$

The increment of the strain  $\Delta\varepsilon_{xm}$  in the  $x$  direction due to the increase in compressive force  $\Delta F$  (where  $\Delta F = F_2 - F_1$ ) was measured. The force  $F_1$  was about 10 %, and the force  $F_2$  about 40 % of the failure force. The modulus  $E_{xg}$  of the glue line made of the adhesive Racoll Express 25 was taken from the literature, similarly like other elastic constants of the glue line.

An expression for the Young's modulus  $E_y$  of the board layer in the  $y$  direction takes the form similar to Eq. (15). The modulus  $E_{ym}$  of the specimen was determined in a similar way to the modulus  $E_{xm}$ .

#### Determination of the Young's modulus $E_z$

The stresses owing to the action of the compressive force in the  $z$  direction (Fig. 7b) are the same in each ply of the specimen.

$$\sigma_z = \sigma_{zg} = \sigma_{zm} \quad (17)$$

where:  $\sigma_z$  and  $\sigma_{zg}$  are the stresses in the board layer and glue line, respectively, and  $\sigma_{zm}$  is the stress in the specimen segment contained into the measurement base of the gauge. These stresses, according to the Hooke's law, are given by

$$\sigma_z = \varepsilon_z E_z, \quad \sigma_{zg} = \varepsilon_{zg} E_{zg}, \quad \sigma_{zm} = \varepsilon_{zm} E_{zm} \quad (18)$$

where:  $\varepsilon_z$  and  $\varepsilon_{zg}$  are the strains in the  $z$  direction of the board layer and glue line, respectively,  $\varepsilon_{zm}$  is the measured strain in the  $z$  direction,  $E_z$  and  $E_{zg}$  are the Young's moduli in the  $z$  direction of the board layer and glue line, respectively, and  $E_{zm}$  is the Young's modulus in the  $z$  direction of the specimen segment contained into the measurement base of the gauge. The shortening of the measurement base is the sum of the shortenings of board layers and glue lines contained into this base:



$$\varepsilon_{zm} b_v = \varepsilon_z (b_v - t_g) + \varepsilon_{zg} t_g \tag{19}$$

where:  $b_v$  is the measurement base (Fig. 7b) and  $t_g$  is the sum of the thicknesses of the glue lines contained into this base.

Combining Eqs. (17) – (19) gives the result:

$$\frac{b_v}{E_{zm}} = \frac{b_v - t_g}{E_z} + \frac{t_g}{E_{zg}} \tag{20}$$

which can be rearranged to an expression for the modulus of elasticity of the board layer, the face and core ones, in the z direction:

$$E_z = \frac{E_{zm} E_{zg}}{E_{zg} \frac{b_v}{b_v - t_g} - E_{zm} \frac{t_g}{b_v - t_g}} \tag{21}$$

The modulus  $E_{zm}$  was determined similarly like the modulus  $E_{xm}$ .

**Determination of the Poisson’s ratios  $\nu_{xy}$  and  $\nu_{yx}$**

Due to the assumed method of loading and measuring deformations of the particleboard layer specimens, the glue lines did not affect the results of determination of the Poisson’s ratios  $\nu_{xy}$  and  $\nu_{yx}$  of the particleboard layers. The ratio  $\nu_{xy}$  was calculated by

$$\nu_{yx} = \left| \frac{\Delta \varepsilon_{xm}}{\Delta \varepsilon_{ym}} \right| \tag{22}$$

The increments of the passive  $\Delta \varepsilon_{ym}$  and active  $\Delta \varepsilon_{xm}$  strains corresponding to the increment of the force  $\Delta F$  were measured.

The Poisson’s ratio  $\nu_{yx}$  was calculated by

$$\nu_{yx} = \left| \frac{\Delta \varepsilon_{xm}}{\Delta \varepsilon_{ym}} \right| \tag{23}$$

on the basis of the measured increments of the passive  $\Delta \varepsilon_{xm}$  and active  $\Delta \varepsilon_{ym}$  strains.

**Determination of the Poisson’s ratios  $\nu_{xz}$  and  $\nu_{yz}$**

The strains in the z direction owing to the action of a load in the x direction (Fig. 7a) are given by

$$\varepsilon_z = -\nu_{xz} \varepsilon_x, \quad \varepsilon_{zg} = -\nu_{xzg} \varepsilon_{xg}, \quad \varepsilon_{zm} = -\nu_{xzm} \varepsilon_{xm} \tag{24}$$

where:  $\varepsilon_z$  and  $\varepsilon_{zg}$  are the strains of the board layer and glue line, respectively,  $\varepsilon_{zm}$  is the measured strain,  $\nu_{xz}$  and  $\nu_{xzg}$  are the Poisson’s ratios of the board line and glue line, respectively, and  $\nu_{xzm}$  is the Poisson’s ratio of the specimen segment contained into the measurement base of the gauge. The elongation of the measurement base is the sum of the elongations of board layers and glue lines contained into this base:

$$\varepsilon_{zm} b_h = \varepsilon_z (b_h - t_g) + \varepsilon_{zg} t_g \tag{25}$$

where:  $b_h$  is the measurement base (Fig. 7a) and  $t_g$  is the sum of the thicknesses of the glue lines contained into this base.

Combining Eqs. (24), (25) and (9) gives:

$$v_{xzm}b_h = v_{xz}(b_h - t_g) + v_{xzg}t_g \quad (26)$$

which can be rearranged to an expression for the Poisson's ratio  $v_{xz}$  of the board layer:

$$v_{xz} = v_{xzm} \frac{b_h}{b_h - t_g} - v_{xzg} \frac{t_g}{b_h - t_g} \quad (27)$$

The ratio  $v_{xzm}$  was calculated by

$$v_{xzm} = \left| \frac{\Delta \varepsilon_{zm}}{\Delta \varepsilon_{xm}} \right| \quad (28)$$

on the basis of the measured increments of the passive  $\Delta \varepsilon_{zm}$  and active  $\Delta \varepsilon_{xm}$  strains.

An expression for the Poisson's ratio  $v_{yz}$  of the board layer takes the form similar to Eq. (27).

The ratio  $v_{yzm}$  was calculated similarly like the ratio  $v_{xzm}$  on the basis of the measured increments of the passive  $\Delta \varepsilon_{zm}$  and active  $\Delta \varepsilon_{ym}$  strains.

#### Determination of the Poisson's ratios $v_{zx}$ and $v_{zy}$

Considering the passive and active strains owing to the action of a load in the  $z$  direction (Fig. 7b), one can derive the following expression:

$$\frac{v_{xzm}a}{E_{zm}} = \frac{v_{zx}(a - t_g)}{E_z} + \frac{v_{zxg}t_g}{E_{zg}} \quad (29)$$

which can be rearranged to a formula for the Poisson's ratio  $v_{zx}$  of the board layer:

$$v_{zx} = v_{xzm} \frac{E_z}{E_{zm}} \frac{a}{a - t_g} - v_{zxg} \frac{E_z}{E_{zg}} \frac{t_g}{a - t_g} \quad (30)$$

where:  $a$  is the gauge width and  $t_g$  is the sum of the thicknesses of the glue lines contained into this width. The ratio  $v_{zxm}$  was calculated by

$$v_{zxm} = \left| \frac{\Delta \varepsilon_{xm}}{\Delta \varepsilon_{zm}} \right| \quad (31)$$

on the basis of the measured increments of the passive  $\Delta \varepsilon_{xm}$  and active  $\Delta \varepsilon_{zm}$  strains.

A formula for the Poisson's ratio  $v_{zy}$  of the board layer takes the form similar to Eq. (30). The ratio  $v_{zym}$  was calculated similarly like the ratio  $v_{xzm}$  on the basis of the measured increments of the passive  $\Delta \varepsilon_{ym}$  and active  $\Delta \varepsilon_{zm}$  strains.

#### Determination of the shear modulus $G_{xy}$

The following relation for the shear moduli in the  $xy$  plane can be derived in a similar manner a Eq. (14) for the Young's moduli in the  $x$  direction:

$$G_{xym}d = G_{xy}(d - d_g) + G_{xyg}d_g \quad (32)$$

where:  $G_{xy}$  and  $G_{xyg}$  are the shear moduli in the  $xy$  plane of the board layer and glue line, respectively, and  $G_{xym}$  is the shear modulus in the  $xy$  plane of the specimen as the layered system consisting of board layers and glue lines.

Rearrangement leads to a following formula for the shear modulus of the board layer:

$$G_{xy} = G_{xym} \frac{d}{d-d_g} - G_{xyg} \frac{d_g}{d-d_g} \quad (33)$$

The modulus  $G_{xym}$  was calculated by Eq. (34) which was obtained from Eq. (8) substituting  $i = x$  and  $j = y$

$$G_{xym} = \frac{E_{xm}^*}{2(1 + \nu_{xym}^*)} \quad (34)$$

The Young's modulus  $E_{xm}^*$

$$E_{xm}^* = \frac{\Delta F}{wd\Delta\varepsilon_{xm}^*} \quad (35)$$

and the Poisson's ratio  $\nu_{xym}^*$

$$\nu_{xym}^* = \left| \frac{\Delta\varepsilon_{ym}^*}{\Delta\varepsilon_{xm}^*} \right| \quad (36)$$

where: calculated on the basis of the measured increments of the active  $\Delta\varepsilon_{xm}^*$  and passive  $\Delta\varepsilon_{ym}^*$  strains in the directions forming angles of  $45^\circ$  to the x and y axes (Fig. 8a) owing to the action of the force increment  $\Delta F$ .

### Determination of the shear moduli $G_{xz}$ and $G_{yz}$

Considering the shear strains in the xz plane one can obtain the following relation for the shear moduli in this plane:

$$\frac{b}{G_{xzm}} = \frac{b-t_g}{G_{xz}} + \frac{t_g}{G_{xzg}} \quad (37)$$

where:  $b$  is the measurement base,  $G_{xz}$  and  $G_{xzg}$  are the shear moduli in the xz plane of the board layer and glue line, respectively, and  $G_{xzm}$  is the shear modulus in the xz plane of the specimen part onto which the gauges were placed.

Rearrangement leads to a formula for the shear modulus of the board layer:

$$G_{xz} = \frac{G_{xzm}G_{xzg}}{G_{xzg} \frac{b}{b-t_g} - G_{xzm} \frac{t_g}{b-t_g}} \quad (38)$$

The modulus  $G_{xzm}$  was calculated by Eq. (39) which was obtained from Eq. (8) substituting  $i = z$  and  $j = x$

$$G_{xzm} = \frac{E_{zm}^*}{2(1 + \nu_{xz}^*)} \quad (39)$$

The Young's modulus  $E_{zm}^*$

$$E_{zm}^* = \frac{\Delta F}{wd\Delta\varepsilon_z^*} \quad (40)$$

and the Poisson's ratio  $\nu_{zxm}^*$

$$\nu_{zxm}^* = \frac{|\Delta \varepsilon_{xm}^*|}{|\Delta \varepsilon_{zm}^*|} \quad (41)$$

where: calculated on the basis of the measured increments of the active  $\Delta \varepsilon_{zm}^*$  and passive  $\Delta \varepsilon_{xm}^*$  strains in the directions forming angles of  $45^\circ$  to the z and x axes (Fig. 8b), corresponding to the increment of the force  $\Delta F$ .

An expression for the shear modulus  $G_{yz}$  of the board layer in the yz plane takes the form similar to Eq. (38). The shear modulus  $G_{yzm}$  in the yz plane of the specimen part onto which the gauges were placed was determined similarly like the modulus  $G_{xzm}$ .

## RESULTS

The elastic constants of the layers of the tested particleboard were calculated on the basis of the measured strains of the layer specimens and the assumed elastic properties of the glue lines made of the adhesive Racoll Express 25. It was assumed that the glue lines are isotropic and the following values of their elastic constants were taken from Smardzewski (2002):  $E_{ig} = 466$  MPa ( $i = x, y, z$ ),  $\nu_{ij} = 0.29$  ( $i, j = x, y, z, i \neq j$ ),  $G_{ij} = 181$  MPa ( $i, j = x, y, z; i \neq j$ ). Microscopic measurements proved that the mean thickness of almost every glue line was less than  $100 \mu\text{m}$ , therefore the thickness of  $0.1$  mm was assumed for calculation. The mean values of elastic constants of the face and core layers of the particleboard are listed in Tab. 1.

Tab. 1: Calculated elastic constants of particleboard layers

Elastic constant	Face layer		Core layer	
$E_x$ (MPa)	4480	(532)	1820	(161)
$E_y$ (MPa)	3760	(469)	1470	(137)
$E_z$ (MPa)	454	(60)	231	(13)
$G_{xy}$ (MPa)	1670	(88)	692	(63)
$G_{xz}$ (MPa)	345	(28)	200	(18)
$G_{yz}$ (MPa)	338	(15)	171	(12)
$\nu_{xy}$	0.252	(0.047)	0.323	(0.061)
$\nu_{yx}$	0.206	(0.027)	0.249	(0.031)
$\nu_{xz}$	0.374	(0.069)	0.334	(0.051)
$\nu_{yz}$	0.346	(0.051)	0.307	(0.054)
$\nu_{zx}$	0.040	(0.007)	0.041	(0.007)
$\nu_{zy}$	0.045	(0.010)	0.049	(0.006)

The obtained elastic constants should satisfy Eqs. (3). The values of ratios for these equations for both layers are given in Tab. 2. They are not the same for a given plane and layer, however, their relative differences are rather small, amount to a few percent. This approximate satisfaction of Eqs. (3) is a positive verification of the assumed model of orthotropy of elastic properties of particleboard layers.

For both the face and core layers the modulus  $E_y$  in the perpendicular direction is, as expected, lower than the modulus  $E_x$  in the mat forming direction. The modulus  $E_y$  amounts to 84 and 81 %

of the modulus  $E_x$  for the face and core layers, respectively. Similar relations between the moduli  $E_y$  and  $E_x$  determined in bending the particleboard layers were obtained by the authors in the previous paper (Wilczyński and Kociszewski 2007). The modulus  $E_z$  in the direction perpendicular to the layer plane is very low amounting only to 10 and 13 % of the modulus  $E_x$  for the face and core layers, respectively. This comparison proves strong anisotropy of the layers in the  $xz$  plane, the plane perpendicular to the plane of layers. The moduli  $G_{xz}$  and  $G_{yz}$  in the planes perpendicular to the layer plane are much smaller than the modulus  $G_{xy}$  in the layer plane. They amount only to 21 and 20 % of the modulus  $G_{xy}$  for the face layer, and to 29 and 25 % of the  $G_{xy}$  for the core layer.

Tab. 2: Comparison of the quotients of Poisson's ratio to Young's modulus for three orthogonal planes of particleboard layers

Plane	Face layer			Core layer		
	$\frac{\nu_{ij}}{E_i}$	$\frac{\nu_{ji}}{E_j}$	Relative difference (%)	$\frac{\nu_{ij}}{E_i}$	$\frac{\nu_{ji}}{E_j}$	Relative difference (%)
$xy$	$5.63 \times 10^{-5}$	$5.48 \times 10^{-5}$	2.7	$1.77 \times 10^{-4}$	$1.69 \times 10^{-4}$	4.5
$xz$	$8.35 \times 10^{-5}$	$8.81 \times 10^{-5}$	-5.5	$1.84 \times 10^{-4}$	$1.77 \times 10^{-4}$	3.8
$yz$	$9.20 \times 10^{-5}$	$9.91 \times 10^{-5}$	-7.7	$2.09 \times 10^{-4}$	$2.12 \times 10^{-4}$	-1.4

The Young's moduli of the core layer range from 39 to 51 % of those of the face layer. The shear moduli of the core layer range from 41 to 58 % of those of the face layer. The mean density of the core layer is 61 % of that of the face layer. The relative differences between elastic moduli of the core and face layers, particularly between the Young's moduli, are greater than the difference between the densities of these layers. One can conclude that the elastic moduli of the particleboard layers are not proportionate to their densities.

The Poisson's ratios  $\nu_{xy}$  and  $\nu_{yx}$  related to the layer plane are over 20 % greater for the core than for the face layer. On the contrary, the Poisson's ratios  $\nu_{xz}$  and  $\nu_{yz}$  related to the transverse layer planes are over 10 % greater for the face layer. The ratios  $\nu_{zx}$  and  $\nu_{zy}$  are slightly greater for the core layer.

## DISCUSSION

The aim of this paper was to present the universal method of determining elastic constants of the particleboard layers, consisting in compressing the block test specimens and measuring their strains by means of electric resistance gages. The specimens were glued from core or face layers separated from the board. The elastic properties of the glue lines were taken into account to calculate the layer elastic constants. The employed method is labour-consuming, however, it has turned out to be effective, its main advantage being the possibility to determine a set of elastic constants, i.e., Young's moduli, Poisson's ratios and shear moduli.

Several studies have been made to learn elastic properties of particleboard layers. Different methods for obtaining selected elastic constants were employed. To determine the Young's modulus of the layers in the direction parallel to the layer plane, the following methods were used: a method of tension of the layer separated from particleboard (Keylwerth 1958, Geimer et al. 1975), an indirect method consisting in comparing the tensile (Dueholm 1976) or bending (May 1983) stiffness of the raw and sanded particleboard specimens, and a method of dynamic mechanical analysis (Bucur et al. 1998). The shear modulus in the layer plane was determined by a plate twisting test (Keylwerth 1958) and the shear modulus in the plane perpendicular to the layer was determined by

an interlaminar shear test (Geimer et al. 1975).

Another way of evaluating the elastic properties of particleboard layers was employed by Wong et al. (2003). They assumed that particleboard consists of plies of homo-profiles boards with various densities and determined elastic properties of such boards laboratory fabricated. Three elastic parameters were determined: the Young's modulus parallel to the board plane by a static bending test, and the Young's modulus perpendicular to the board plane and the shear modulus in the board plane by dynamic methods.

Wilczyński and Kociszewski (2007) determined the layer moduli of elasticity for three directions: the mat forming direction, the direction perpendicular to it, and the direction perpendicular to the layer plane by a bending test, using the specimens glued from layers separated from boards.

The full set of elastic constants of the face and core layers of typical commercial particleboard were determined by the method presented in this study. Knowledge of these constants can enable a thorough strength analysis of structural members made of particleboards. Particularly, the layer elastic constants data can be useful for finite element analysis in the case in which orthotropic 3D elements within each layer are employed, for example when stresses and deformations at the place of the embedment of connectors are to be analysed.

## CONCLUSION

The universal method presented in this paper, consisting in compressing the block specimens glued from strips of layers separated from particleboard, and measuring specimen deformations by the electric resistance strain gauge technique, enables to determine the set of elastic constants: three Young's moduli, six Poisson's ratios and three shear moduli for both the face and core layers of particleboard.

The results of investigation of the elastic properties of the layers of typical commercial three-layer particleboard carried out using this method confirmed the assumption that particleboard layers are orthotropic materials.

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