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EXAMINATION OF THE EDGEWISE SHEAR MODULUS OF WOOD MEASURED BY DYNAMIC SQUARE-PLATE TWIST

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ABSTRACT

The edgewise shear modulus in the longitudinal-radial plane (G_{LR}) obtained by dynamic square-plate twist tests of solid wood using specimens with various thicknesses was experimentally and numerically analyzed. A free-free flexural vibration test of the beam specimen was also conducted, and the results were compared with those of dynamic square-plate twist tests. The value of G_{LR} was effectively obtained by introducing the coefficient obtained from the finite element analyses.

KEYWORDS: Dynamic square-plate twist test, finite element analysis (FEA), flexural vibration test, edgewise shear modulus, approximation equation.

INTRODUCTION

Obtaining reliable shear properties for wood and wood-based materials, including their shear modulus, is essential for ensuring that construction procedures are efficient and cost effective. Various experimental methods are available for determining the shear modulus of solid wood and wood-based materials (Yoshihara 2006). Among these methods, the vibration method is quite effective because the load applied to the specimen is so small that the shear modulus can be measured non-destructively. In particular, the flexural vibration method using a beam specimen is accurate enough to measure the shear modulus, so this method is frequently conducted (Hearmon 1958, 1966, Nakao et al. 1985a, Sobue 1988, Kubojima et al. 1996, 1997, Brancheriau and Bailleres 2002). In this method, however, the specimen should be slender, and the method is thus not appropriate for measuring the shear modulus of a plate specimen. In the practical operation, there are many occasions where solid wood and wood-based materials with a plate shape are used, such as for walls, floors, and the soundboards of musical instruments, so it is desirable to measure the shear modulus of material with a plate shape. The dynamic square-plate

twist method makes use of plate specimens, and therefore provides shear modulus of solid wood and wood-based materials that are appropriate for practical construction applications. In addition, this method is advantageous because the test is simple to conduct. Nakao and Okano (1987) and Yoshihara (2009) conducted dynamic square-plate twist tests on plywood and medium-density fiberboard. In these studies, it was suggested that the edgewise shear modulus can be measured properly when the thickness/length ratio of the specimen is small enough, because the influence of orthotropy could be effectively reduced. Compared to the wood-based materials, however, solid wood has a high orthotropy which influences the measurement of shear modulus even when the specimen has a large thickness/length ratio (Nakao et al. 1985b). When this drawback is overcome, the shear modulus of solid wood may be appropriately measured by the dynamic square-plate twist method.

In this work, dynamic square-plate twist tests were conducted under various testing conditions. The validity of the method was examined by comparing its results with those of a free-free flexural vibration test of beam specimen and finite element calculations.

Dynamic square-plate twist method

Fig. 1 shows a diagram of the dynamic square-plate twist test. In this study, the x axis is the longitudinal direction, the y axis is the radial/tangential direction and the z axis is the tangential/radial direction of the material. The specimen is suspended by threads along the mid-length of each side, which corresponds to the nodal lines in the first mode of torsional vibration. When a corner of the plate is struck by a hammer, torsional vibration is generated and the edgewise shear modulus G_{xy} is obtained using (Hearmon 1948, Nakao and Okano 1987).

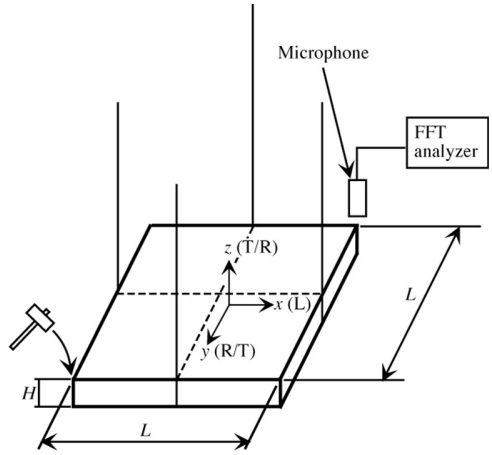

Fig. 1: Scheme of the dynamic square-plate twist test. L, R, and T represent the longitudinal, radial, and tangential directions, respectively.

$$G_{xy} = \beta\rho\left(\frac{L^2}{T} f_t\right)^2 \tag{1}$$

where: L, T, and ρ are the side dimension, thickness, and density of the plate, respectively, and f_t is the resonance frequency for the first torsional vibration mode. The term β is a coefficient given as follows:

$$\beta = \frac{12\pi^2}{\frac{D_{11}}{4D_{66}}\alpha_1 + \frac{D_{22}}{4D_{66}}\alpha_2 + \frac{D_{12}}{2D_{66}}\alpha_3 + \alpha_4} \tag{2}$$

where: D11-D66 are obtained using the Young's moduli in the x and y directions E_x and E_y, respectively, and major Poisson's ratio in the xy plane v_{xy}, and G_{xy} as follows:

$$\begin{cases} D_{11} = \dfrac{E_x T^3}{12\left(1 - v_{xy}^2 \dfrac{E_y}{E_x}\right)} \\ D_{22} = \dfrac{E_y T^3}{12\left(1 - v_{xy}^2 \dfrac{E_y}{E_x}\right)} \\ D_{12} = v_{xy} D_{22} \\ D_{66} = \dfrac{G_{xy} T^3}{12} \end{cases} \tag{3}$$

where: α_1-α_4 are the parameters determined from the vibration mode and boundary condition.

The complexity of Eq. (2) indicates the inconvenience for obtaining G_{xy} with a theoretical rigorousness. To determine G_{xy} practically, some approximation is therefore introduced into β. Several previous studies on plate vibration suggested that β is approximated into 0.822 (Hearmon 1960) or 0.9 (Leissa 1969, Nakao and Okano 1987) for thin material. When the thickness of the material is relatively large, however, β was suggested to depend on the specimen configuration because of the deflection caused by shearing force applied on side surfaces (Nakao et al. 1985a, Nakao and Okano 1987).

Finite element analysis

Three-dimensional finite element analyses were conducted independently of the experimental shear modulus measurements. The finite element program used was ANSYS Version 5.7, which is a library program of the Information Initiative Center of Hokkaido University. Fig. 2 shows the finite element meshing used for the square-plate twist specimen. The finite element models employed eight-noded brick elements. The length in the x and y directions was 140 mm, whereas the length in the z direction varied from 3 to 15 mm at an interval of 3 mm. The elastic properties needed for the calculations are listed in Tab. 1. The Young's moduli in the L, R, and T directions are designated E_L, E_R, and E_T, respectively. The shear moduli in the LR, LT, and RT planes are designated G_{LR}, G_{LT}, and G_{RT}, respectively. The Poisson's ratios in the LR, LT, and RT planes are designated v_{LR}, v_{LT}, and v_{RT}, respectively. The properties were taken from Hearmon (1958), the Wood Industry Handbook (2004), and Valentin and Adjanohoun (1992).

In previous studies, the degree of orthotropy was evaluated from the relationship between $E_x/2G_{xy}$ - v_{xy} and E_x/E_y (Kageyama et al. 1991, Yoshihara 2010a,b). Although this relationship may be effective for two-dimensional analysis, it may not be appropriate for three-dimensional analysis. In this study, this relationship is therefore extended as follows:

$$\begin{cases} p_{\mathrm{I}} = \dfrac{1}{6} \displaystyle\sum_{i,j=\mathrm{L,R,T}} \dfrac{E_j}{E_i} & (i \neq j) \\ p_{\mathrm{II}} = \dfrac{1}{6} \displaystyle\sum_{i,j=\mathrm{L,R,T}} \left(\dfrac{E_i}{2G_{ij}} - v_{ij}\right) & (i \neq j) \end{cases} \tag{4}$$

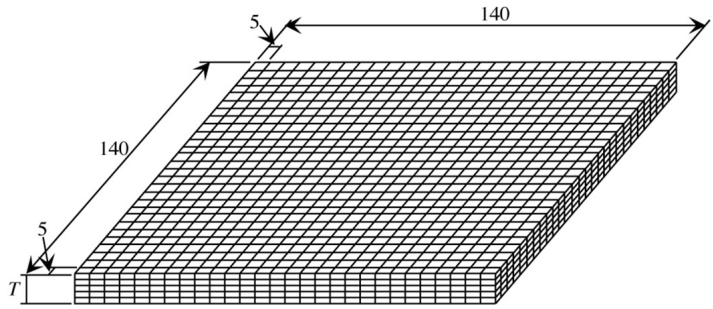

Fig. 2: Finite element model used for the dynamic square-plate twist test analysis. Unit = mm.

Tab. 1: Elastic properties used for the finite element calculations.

		Density ($kg.m^{-3}$)	Young's modulus (GPa)			Poisson's ratio			Shear modulus (GPa)		
Species	**Code**	**ρ**	E_L	E_R	E_T	ν_{LR}	ν_{LT}	ν_{RT}	G_{LR}	G_{LT}	G_{RT}
Softwood											
Spruce[1)]	A	500	16.6	0.85	0.43	0.36	0.52	0.43	0.63	0.84	0.037
Yezo spruce[2)]	B	390	10.8	0.83	0.44	0.40	0.60	0.65	0.54	0.44	0.020
Scots pine[1)]	C	550	16.3	1.10	0.57	0.42	0.51	0.68	1.16	0.68	0.066
Japanese red pine[2)]	D	590	16.4	1.30	0.90	0.43	0.37	0.63	1.18	0.91	0.079
Douglas fir[1)]	E	480	15.7	1.06	0.78	0.29	0.45	0.39	0.88	0.88	0.088
Standard softwood[3)]	F	450	13.1	1.00	0.64	0.38	0.42	0.49	0.86	0.75	0.084
Hardwood											
Balsa[1)]	G	100	2.4	0.11	0.038	0.23	0.49	0.66	0.12	0.085	0.014
Japanese birch[2)]	H	620	16.3	1.11	0.62	0.49	0.43	0.78	1.18	0.91	0.19
Oak[1)]	I	660	5.3	2.14	0.97	0.33	0.50	0.64	1.29	0.76	0.39
Walnut[1)]	J	590	11.2	1.19	0.63	0.49	0.63	0.72	0.96	0.70	0.23
Red Lauan[2)]	K	510	11.5	0.97	0.41	0.38	0.60	0.85	0.46	0.27	0.16
Standard hardwood[3)]	L	650	14.4	1.81	1.0	0.39	0.46	0.68	1.26	0.97	0.37

L, R and T represent the longitudinal, radial, and tangential directions, respectively. Properties are adopted from 1: Hearmon (1958), 2: Wood Industry Handbook (2004), 3: Valentin and Adjanohoun (1992). Latin names of the species: Spruce: *Picea sp.*, Yezo spruce: *Picea jezoensis*, Scots pine: *Pinus sylvestris*, Japanese red pine: *Pinus densiflora*, Douglas fir: *Pseudotsuga menziesii*, Balsa: *Ochroma lagopus*, Japanese birch: *Fagus crenata*, Oak: *Quercus* sp., Walnut: *Juglans* sp., Red Lauan: *Shorea teysmanniana*.

Fig. 3 shows the relationship between p_I and p_{II} for various wood species, adopted from Hearmon (1958), the Wood Industry Handbook (2004), and Valentin and Adjanohoun (1992). For isotropic material, $p_I = p_{II} = 1$. The species in Tab. 1 were selected because of their differing values of p_I and p_{II} (Fig. 3).

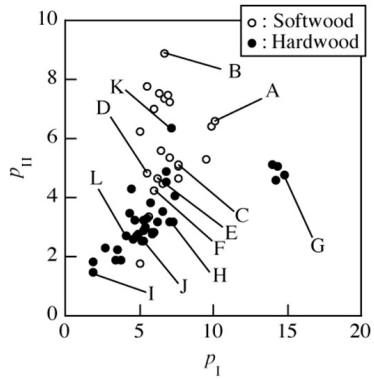

Fig. 3: Relationship between p_I and p_{II} obtained from Eq. (4). A-K: See Tab. 1. The data represented by white circles are adopted from Hearmon (1958), the Wood Industry Handbook (2004), and Valentin and Adjanohoun (1992).

Modal analyses were conducted and the resonance frequency for the first torsional vibration mode ft was extracted. The value of β was calculated by substituting G_{xy} input into the finite element program and f_t into Eq. (1).

MATERIAL AND METHODS

Materials

Sitka spruce (*Picea sitchensis*), Douglas fir (*Pseudotsuga menziesii*), Japanese birch (*Fagus crenata*), and mersawa (*Anisoptera* sp.) lumbers were investigated. The densities in kg.m^{-3} at 12 % moisture content (MC) where: spruce 375 ± 5, Douglas fire 640 ± 12, birch 610 ± 8, and mersawa 570 ± 10 kg.m^{-3} respectively. These lumbers had no defects such as knots or grain distortions, so the specimens cut from them could be regarded as "small and clear". The lumbers were stored for approximately five years in a room at a constant 20°C and 65 % RH before the test, and the specimens were confirmed to be in an air-dried condition. These conditions were maintained throughout the tests. The equilibrium MC condition was approximately 12 %. Seven specimens were tested for each test condition described below.

Dynamic square-plate twist tests

The specimen dimensions were 140 (L) x 140 (R) x 3-15 (T) mm: specimen thickness *T* varied at intervals of 3 mm. It was difficult to prepare the specimens with wider surface in the LT plane, although finite element analyses using the model with the wider LT plane were conducted, because of the volume limitations of the test materials. Therefore, the test conditions were restricted as described.

The specimen was suspended by threads along the mid-length of the side, as shown in Fig. 1. Torsional vibration was generated by striking a corner of the plate, and the resonance frequency of the first torsional vibration mode f_t was measured and analyzed by a Fast Fourier transform (FFT) analysis program. Using the results obtained from the finite element analyses, the shear modulus in the LR plane G_{LR} was analyzed. The details are described below.

Flexural vibration tests

Independently of the dynamic square-plate tests, G_{LR} was determined by the flexural vibration tests, the validity of which was satisfactorily verified in several previous works (Hearmon 1958, 1966, Nakao et al. 1985b, Kubojima et al. 1996, 1997).

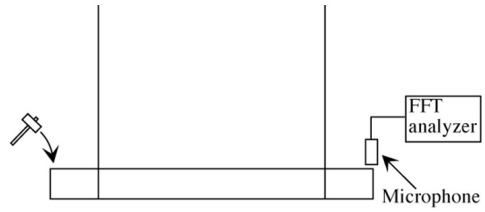

Fig. 4: Diagram of the free-free flexural vibration test.

A beam specimen with the dimensions of 350 (L) x 30 (R) x 12 (T) mm was prepared. The specimen was suspended by threads at the nodal positions of the free-free resonance vibration mode f_n and excited in the thickness (R) direction with a hammer (Fig. 4). First- to the fourth-mode resonance frequencies were measured and analyzed by the FFT analysis program. G_{LR} was calculated by Hearmon's iteration method (Hearmon 1958), which is based on an approximation of Timoshenko's flexural vibration solution that was developed by Goens (1931). In Hearmon's method, multiple resonance frequencies for flexural vibration modes are measured and X and Y corresponding to each mode are calculated as follows (Hearmon 1958):

$$\begin{cases} X = \dfrac{4\pi^2 \rho L^2 f_n^2}{m_n^4}\left[-2m_n F(m_n) + m_n^2 F^2(m_n)\right] \\ Y = \dfrac{4\pi^2 \rho A L^4 f_n^2}{m_n^4 I}\left[1 + \dfrac{6m_n F(m_n) I}{L^2 A} + \dfrac{m_n^2 F^2(m_n) I}{L^2 A} - \dfrac{4\pi^2 \rho I f_n^2}{k G_{LR} A}\right] \end{cases} \tag{5}$$

where: ρ is the density of the beam, L is the length of the beam, A is the cross-sectional area, I is the secondary moment of inertia, and k is the shear correction factor, which is 5/6 that for a beam with a rectangular cross-section. The coefficients m_n and $F(m_n)$ that correspond to the resonance mode are

$$\begin{cases} m_1 = 4.730 \\ m_2 = 7.853 \\ m_n = \dfrac{(2n+1)\pi}{2} \quad (n \geq 3) \end{cases} \tag{6}$$

and

$$\begin{cases} F(m_1) = 0.9825 \\ F(m_2) = 1.0008 \\ F(m_n) = 1 \quad (n \geq 3) \end{cases} \tag{7}$$

The plot of X vs Y for each mode was regressed into the linear function $Y = S - RX$, and G_{LR} was to be S/kR. Initially, a virtual value of G_{LR} was substituted into Y of Eq. (5), and the refined value of G_{LR} as S/kR was substituted into Y again and the procedure was repeated to give the regressed value of G_{LR}.

RESULTS AND DISCUSSION

Fig. 5 shows the relationships between the coefficient β and the thickness/length ratio T/L obtained by the finite element analyses for the twelve wood species shown in Tab. 1. The results of the finite element analyses conducted here reveal that β increases with increasing T/L, and that it can be represented by a linear relationship with the intercept of 0.9 as follows:

$$\beta = 0.9 + Q\frac{T}{L} \tag{8}$$

As described above, the deflection caused by shearing force in the thickness direction increases with increasing *T/L* (Nakao et al. 1985a, Nakao and Okano 1987). When comparing the results for softwood and hardwood, Q obtained from softwood is larger than that obtained from hardwood. However, the influence of a wider surface (LR or LT) is not significant.

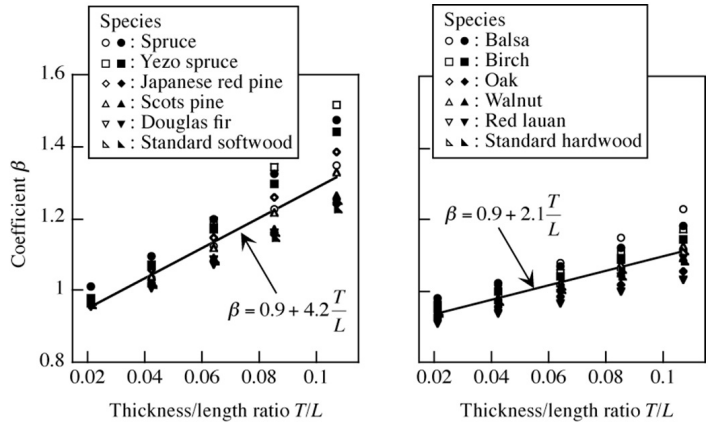

Fig. 5: Coefficient β vs specimen thickness/length ratio T/L.

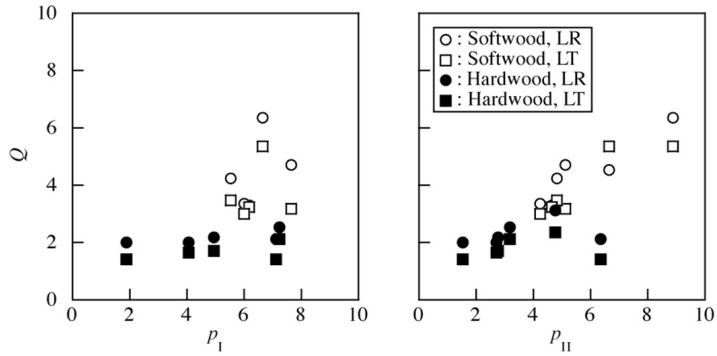

Fig. 6: Dependence of the slope Q in E_q. (8) on the parameters p_I and p_{II}.

Fig. 6 shows plots of slope Q in E_q. (8) vs parameters p_I and p_{II} obtained by finite element analysis. Q increases significantly with increasing p_{II}. In contrast, the influence of p_I is less

significant than that of p_{II}. As described above, the out-plane deflection caused by the shearing force influences β. The shear moduli in the side surfaces are contained in p_{II}, so the slope Q included in β is influenced from p_{II}.

As shown in Fig. 5, β would be derived for softwood in spite of rough approximation as follows:

$$\beta = 0.9 + 4.2\frac{T}{L} \tag{9}$$

For hardwood, β would be derived as follows:

$$\beta = 0.9 + 2.1\frac{T}{L} \tag{10}$$

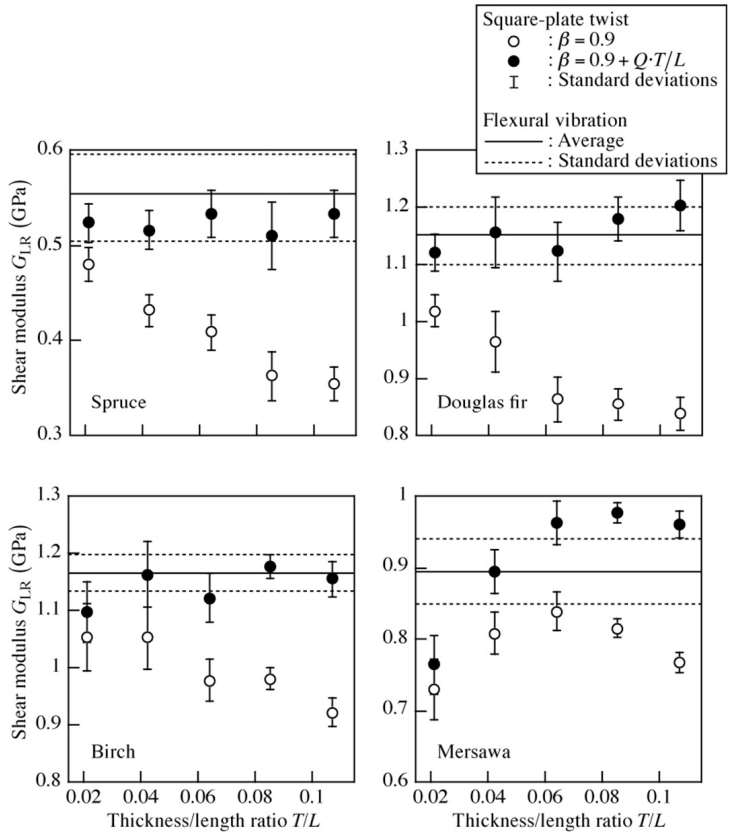

Fig. 7: Shear modulus G_{LR} vs. specimen thickness ratio T/L obtained from the dynamic square-plate twist and flexural vibration methods.

Fig. 7 shows plots of the edgewise shear modulus G_{LR} vs the specimen thickness/length ratio T/L, obtained by dynamic square-plate twist and flexural vibration tests. When substituting β = 0.9 into Eq. (1), the G_{LR} decreases significantly with increasing *T/L* except for the results

obtained from mersawa. In contrast, when using Eqs. (9) and (10) for the data of softwood and hardwood species, respectively, G_{LR} varies less with *T/L* than that using $\beta = 0.9$, and is close to that obtained by the flexural vibration test. It may therefore be effective to use Eqs. (9) and (10) for measuring the edgewise shear modulus of softwood and hardwood, respectively. For mersawa, however, the relationships between G_{LR} and *T/L* are different from those obtained using the other species. To establish the dynamic square-plate twist method, further research should be undertaken under various test conditions using various species.

CONCLUSIONS

A test of Sitka spruce, Douglas fir, Japanese birch, and mersawa was conducted to obtain the shear modulus in the longitudinal-radial plane G_{LR} using specimens with various thicknesses. The dynamic square-plate twist test method was experimentally and numerically analyzed. In the latter analysis, 3D finite element analysis was conducted and the shear modulus was calculated with conducting the modal analysis for the models with various thicknesses. In addition to the dynamic square-plate twist test, the free-free flexural vibration test was performed, and the results were compared with those of the dynamic square-plate twist tests.

We have interpreted our test results to mean that the shear modulus of several species can be obtained effectively by introducing the coefficient β, which is dependent on the thickness/length ratio of the specimen, in spite of the rough approximation. To increase the accuracy of the measurement, however, further research should be undertaken using various specimens.

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