THE DYNAMIC MEASUREMENT OF CONTINUOUS BENDING RIGIDITY OF CURVED PLYWOOD STRIPS

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ABSTRACT

It was offered to measure the continuous bending rigidity (the parameter of quality) of curved plywood strips (products) using continuous bending technique. The strip moves over the production line on the conveyor belt where it is bent when it is placed over the recessed path and the load force and the bending amplitude is measured. The bending rigidity and its variation along the strip are calculated. The prototype of the measurement system was implemented in the production line of the curved plywood strips. Relative expanded measurement uncertainty of the dynamic measurement of continuous bending rigidity is $U_{x} = \pm 0.012$ or 1.2%.

KEYWORDS: Curved plywood strips, continuous bending rigidity, dynamic measurement.

INTRODUCTION

During the recent years the global furniture industry introduced the wide use of the curved plywood strips with the grain (fibre) going the same way which are used as the load-supporting and dampening element.

The conceptions of the terms of the curved plywood strips are usually defined individually by assessing the area of application and the design peculiarities (Plywood design specification 1990).

Methods defined by standards and based on mechanical tests are used to determine the properties of the multilayer wood sheet materials and the analyzed sample is destroyed in most cases of the testing (ASTM D 3043, ISO DIS 8982) methods of non-destructive control such as acoustic emission, ultrasound and acoustic ultrasound, method of free vibrations and other are also used (Beall 2002, Augutis et al. 2007).

In this work the continuous bending rigidity $S$ of the curved plywood strip (product) is evaluated by assessing the ratio of the load force $F$ to the bending amplitude $d$ (in the range of the linear bending proportionality to the load of the strip) by using the supporting elements at the ends of the strip and by applying the load force in the middle of it.
MATERIAL AND METHODS

In order to verify the bending rigidity of the curved plywood strips both standardized and non-destructive control methods may be applied. In laboratories the standardized methods are usually selected for the measurement of the parameters of strip materials. However, the application of the standardized techniques is often unacceptable when implementing the control of the bending rigidity of the curved plywood strips (product) directly in the production line. Mostly this fact is conditioned by the long control process duration compared to the rate of the production line; on the other hand, it is unacceptable because the product is no more suitable to be used for the commercial purposes after such verification.

The continuous bending method was offered for dynamic measurement of the continuous bending rigidity of the curved plywood strip directly in the production line. Method is based on the loading of the strip using the fixed-location load when the strip moves over the conveyor belt over specially installed deflection as it is illustrated in Fig. 1.

![Fig. 1: The scheme of the strip bending rigidity measurement using the continuous bending technique: 1 – fixed support with the bending angle \( \alpha \), 2 – transporter belt, 3 – tested strip, 4 – pressing wheel, 5 – force measurement transducer, 6 – upper fixed support, \( V \) – strip dragging direction.](image)

It was determined experimentally that there is some certain range of the loads for each type of the curved plywood strips and the strip bending amplitude is linearly related to the load in this range of the loads.

When the strip is dragged over the bent support, its bending amplitude is different in the different areas and the force transducer is impacted with varying force. In order to evaluate the bending rigidity variation we will refer to the model given in Fig. 2.

![Fig. 2: The variation of the strip bending amplitude when it is dragged over the bent support: \( y \) – the bending amplitude of the measured strip; \( F \) – loading force; \( l \) – distance between the supports; \( x \) – strip load displacement in respect of the middle point between the supports.](image)

When the load is applied in the middle between the supports its bending amplitude \( y_a \) in the middle of the strip is calculated as Ziliukas (2004):

\[
\text{(1)}
\]
where:  
- F – load,
- l – distance between supports,
- E – Young’s modulus of the material,
- b – width of the strip,
- h – thickness of the strip.

When the load application point deviates from the middle point of the strip, the strip bending amplitude at the application point of the force is calculated as Žiliukas (2004):

\[
y_a = \frac{F \cdot l^3}{4 \cdot E \cdot b \cdot h^3}
\]  \hspace{1cm} (1)

By using equation (2), the strip bending rigidity at its any point and when varying the load location along the strip can be calculated in the following way:

\[
y(x) = \frac{F \cdot (\frac{l}{2} - x)[3 \cdot l^2 - 4(\frac{l}{2} - x)^2]}{4 \cdot E \cdot b \cdot h^3}
\]  \hspace{1cm} (2)

Strip continuous bending rigidity variation characteristics calculated according to equation (3) is given in Fig. 3.

\[
S(x) = \frac{4 \cdot E \cdot b \cdot h^3}{(0.5 \cdot l - x \cdot (2 \cdot l^2 + l \cdot x)}
\]  \hspace{1cm} (3)

It was determined from the calculation that when the load is applied at the middle zone of the strip, then the strip bending rigidity is practically constant. Therefore the bending rigidity determined in the middle point of the strip by using continuous bending method will match the bending rigidity determined using the standard static bending method.

In case of the curved plywood strips (of the investigated types) it was experimentally determined that the bending angle \( \alpha \) of the support must not exceed 5°, because the Hook’s law is valid up to such strip bending amplitude when measuring rigidity inside the bending range.

In order to evaluate the relation between the force acting upon the force transducer and the strip bending amplitude it is purposeful to use the scheme of the strip movement over the deflection area shown in Fig. 4.
Fig. 4: Strip movement along the conveyor line: $l$ – the length of the measured strip (in this case it is considered to be equal to the distance between the supports when the static measurement is used), $a$ – the strip ordinate under the assumption that the strip is not bent, $b$ – the ordinate of the bent strip, $d$ – abscissa of the middle of the strip, $\alpha$ – deflection angle of the lower fixed support; $c$ – abscissa of the beginning of the strip.

Since the load-imitating support (with the wheel at its end to decrease the friction), at the end of which the force measurement transducer is installed, is fixed stationary, then the force $F$ acting upon the support and at the same time upon the force measurement transducer will be proportional (in the range of the linear dependency between the load and the bending amplitude) to the strip continuous bending rigidity and its momentary bending amplitude $(y_a(x) - y_b)$:

$$F = S(y_a(x) - y_b)$$  \hspace{1cm} (4)

where: $y_b$ – the ordinate of the force transducer and the strip contact point,
$y_a(x)$ – momentary ordinate of the strip surface in case the strip is not bent.

When the strip moves along the conveyor and the support bends the strip, then the momentary ordinate $y_a(x)$ at the point $a$ is calculated as

$$y_a(x) = \frac{2}{l} \left[ \left( \frac{l}{2} \right)^2 - x^2 \right] \sin(\alpha)$$  \hspace{1cm} (5)

For the fixed support of the load we may assume that the ordinate $y_b = 0$; then the force acting upon the force transducer according to (3) and (5) is calculated in the following way

$$F(x) = S \cdot y_a(x) = \frac{8 \cdot E \cdot b \cdot h^3 \cdot \sin \alpha}{l^2} \cdot \frac{l + 2x}{2(2l + x)}$$  \hspace{1cm} (6)

Equation (6) is suitable for the non-curved strip. In case of curved strip, the curvature of the strip should be added to the ordinate $y_a(x)$. Curvature is calculated as

$$y_R(x) = y_{R0} - R \cdot x^2$$  \hspace{1cm} (7)

where: $R$ – curvature radius of the strip,
$y_{R0}$ – curvature of the strip at its middle point.

In this way the force acting upon the force transducer will be $F(x) = S(y_a(x) + y_R(x))$.

When measuring the continuous bending rigidity of the curved plywood strips directly in
the manufacturing line, their real curvature $y_R(x)$ is measured using separate measuring device containing displacement measurement transducer.

Since the output voltage of the force measurement transducer is linearly proportional to the loading force (Žiliukas 2004), the strip continuous bending rigidity can be evaluated according to the value of the output voltage of the force transducer, which is obtained when the middle of the moving strip is placed under the load support (Fig. 2).

Measuring system channel is calibrated using the reference curved strip the bending rigidity of which $S_0$ is measured in static mode.

When the middle of the reference strip moves under the force transducer in the measurement line, the output signal of the transducer $u_0 = k.S_0$, here $k$ – coefficient of proportionality, which assesses the load force and the transducer sensitivity.

When measuring the curved strip of the unknown rigidity, the signal at the transducer output will be $u_x$, therefore the bending rigidity of such strip is calculated as

$$S_x = \frac{u_x}{u_0}S_0$$

(8)

Proportionality coefficient was determined experimentally for each type (reference) of the curved plywood strip.

RESULTS AND DISCUSSION

In the considered case the dimensions of the curved plywood strips verified directly in the manufacturing line are: length: from 400 to 1200 mm; width: from 25 to 80 mm; thickness: from 8 to 12 mm, and the curvature radius of the strips 5000 mm. The movement speed of the curved strip on the transporter belt: from 30 to 70 m.min$^{-1}$.

The proportionality coefficient $k$ was determined experimentally for each type (reference) of the curved plywood strip.

The load force variation along the strip, when measuring curved strips of the same type but different rigidity is shown in Fig. 5.

![Fig. 5: The load force variation along the curved strip: a) when measuring the two strips of the same type of good and poor quality (line with markers); b) when measuring the same strip for ten times (zoomed view).]
Measurement uncertainty analysis

The relative errors of the force measurement, bending amplitude measurement, distance between supports and the force application point deviation from the middle point between the supports measurement are the main components of the uncertainty.

On the basis of (1) and (3) the relative expanded continuous bending rigidity measurement uncertainty $U_{sx}$ is calculated as

$$U_{sx} = 2 \sqrt{\gamma_x^2 + \gamma_0^2 + \frac{2}{3} (\gamma_F^2 + \gamma_d^2 + 9 \gamma_l^2) + \left(\frac{9}{2} \gamma_{\Delta}^2\right)}$$

(9)

where: $\gamma_F$, $\gamma_d$ and $\gamma_l$ – relative measurement errors of the force, bending amplitude and distance between the supports, respectively. We will consider the distributions of these errors of equal probabilities. $\gamma_{\Delta}$ – relative measurement error of the force application point deviation from the center, $\gamma_0$, $\gamma_x$ – relative errors of the force measurement channel, when measuring reference and sample strips, respectively; determined experimentally. Their distributions are considered to be normal.

In the implemented prototype of the measurement system the typical errors of the force transducer of the type U93 used to measure the load force and the FESTO Standard Cylinder with axis controller SPC200 used to measure bending amplitude were respectively (Hottinger Baldwin Messtechnik 2010, Festo 2010): $\gamma_F = \pm 5 \cdot 10^{-3}$, $\gamma_d = \pm 2.2 \cdot 10^{-3}$ and the relative errors of the distance between the supports and the force application point measurement were respectively $\gamma_l = \pm 2 \cdot 10^{-3}$, $\gamma_{\Delta} = \pm 2 \cdot 10^{-3}$.

Components $\gamma_x$, $\gamma_0$ are determined by measuring similar magnitudes under the same conditions therefore the systematic error components are compensated and only random errors remain and furthermore, they are of the similar magnitudes. For this reason it was assumed that $\gamma_x = \gamma_0 = \gamma_e$. Component $\gamma_e$ was determined experimentally by measuring 10 curved plywood strips of 5 types and of good quality ($n = 50$).

Experimental dispersion, strip bending amplitude measuring dispersion and relative error

$$s_{\gamma_0}^2 = \frac{1}{n-1} \sum_{i=1}^{n} (y_{0i} - y_{0\text{mean}})^2 = 5.09 \cdot 10^{-3}$$

(10)

$$\gamma_e = \frac{s_{\gamma_0}}{y_{0\text{mean}}} = 1.18 \cdot 10^{-3}$$

(11)

By evaluating all components, the calculated relative expanded uncertainty of the dynamic measurement of continuous bending rigidity in the manufacturing line was $U_{sx} = \pm 0.012$ or 1.2 %.

CONCLUSIONS

The continuous bending technique was applied to measure the continuous bending rigidity of the curved plywood strips directly in the manufacturing line.

The relative expanded uncertainty of the dynamic measurement of the bending rigidity directly in the manufacturing line is $U_{sx} = 0.012$. 326
When measuring the bending rigidity of the curved plywood strip using continuous bending technique, it is not required to decrease the performance of the manufacturing line. When the type of the tested strips is changed, only the software must be changed.

REFERENCES

5. ISO DIS 8982, 1988: Plywood - Determination of physical and mechanical properties for structural purposes.