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COMPARISON BETWEEN TSAI-WU FAILURE CRITERION AND HANKINSON´S FORMULA FOR TENSION IN WOOD

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ABSTRACT

For wood a material with anisotropic characteristics and strength asymmetry, Hankinson´s formula was the first successful expression to predict the strength in non-parallel orientation to the grains. However, with the emergence of new materials, with directional properties, there was a need to formulate broader failure criteria. One of these criterions is Tsai-Wu's general failure theory for anisotropic materials. This research seeks to investigate the use of this failure criterion for wood, considering its directional properties. The failure parameters of Tsai-Wu criterion were determined from uniaxial tension and compression in specimens of the wood species *Goupia Glabra*, at inclined directions with respect to the fibers (15 °, 30 °, 45 °, 60 °, 75 °), shear and biaxial compression test. The estimates of Tsai-Wu criterion are compared to the values from Hankinson's formula. The results were in general appropriate and thus reveal an interesting way to be followed for future research.

KEYWORDS: Failure criteria, Tsai-Wu criterion, Hankinson´s formula, tensile tests, wood.

INTRODUCTION

Various of the existing criteria present restrictions for its use in anisotropic materials or with directional properties, such as wood, making it important to carry out strength criteria evaluations aiming at estimating the material failure under simple or multiple stresses.

Almost with no exception, according to Bodig and Jayne (1993), the failure criteria were developed for homogeneous isotropic materials, with the hypothesis of linear behavior, stress and deformation relation, until failure. Therefore, its applicability gets limited to materials such as wood and wood composites due to their non-homogeneity and non-elastic behavior.

According to Bodig and Jayne (1993), because of the complexity of the failure phenomena in wood composites the estimates using failure theories have not been totally developed yet. Consequently, empirical methods can be used, which under certain circumstances can be reasonably accurate. Hankinson's formula is one of these methods. Applied frequently to wood, mainly in compression for strength evaluation with inclined fibers, the formula presents the following general situation:

$$\sigma_\theta = \frac{f_0 \cdot f_{90}}{f_0 \cdot \sin^n \theta + f_{90} \cdot \cos^n \theta} \tag{1}$$

where: σ_θ - strength in an inclined orientation in relation to the fibers, f_0 - parallel strength to the fibers, f_{90} - perpendicular strength to the fibers θ - inclination angle of the fibers and n - exponent.

Hankinson's formula, although strictly empirical, has presented values close to the values obtained in wood specimens.

Among the existing failure criteria, the general failure theory developed by Tsai and Wu (1971) for anisotropic materials is one of the most consistent criteria as it presents various advantages when compared to several other existing theories, as presented in item 2.

Considering not only anisotropy but also the material strength assymetry, the general failure criterion from Tsai and Wu can be expressed in the following way:

$$F_i \cdot \sigma_i + F_{ij} \cdot \sigma_i \cdot \sigma_j = 1 \tag{2}$$

where: i, j from 1 to 6 and where F_i and F_{ij} are strength parameters determined through uniaxial tensile, compression and shear tests as well as tensile tests.

With the definition of the strength parameters, it is possible to determine the failure surface for the studied material and predict the strength under various stress conditions.

In this context, this work aims at evaluating Tsai-Wu's failure criterion, used for anisotropic materials, for wood. To do so, tests with wood specimens are carried out to determine the strength parameters of this criterion and the wood strength with inclined fibers. The test results are compared to this failure criterion and also to Hankinson's formula.

Tsai-Wu's criterion

Eq. 2, developed by Tsai and Wu, represents the general failury theory for anisotropic materials. The expanded version of the equation can be observed in Eq. 3.

The various characteristics of the strength criterion proposed by Tsai and Wu are:

-it is a scalar and automatically invariant equation. The interactions between all the stress components are independent from the material properties,

-the strength components are expressed in a tensor, its transformation relationships and the associated invariants are well established.

-The symmetry properties of the strength tensor and the number of null and independent components can be rigorously determined in the same way as the other properties of the anisotropic materials, such as the elasticity matrix.

- by knowing the transformation relationships it is possible to easily rotate the material axes from F_i to $F_{i'}$ and from F_{ij} to $F_{ij'}$, or also change the applied stress from σ_i to σ_i' when one want to study the properties outside the main axes or the modified properties.

$$\begin{aligned}
&F_1 \cdot \sigma_1 + F_2 \cdot \sigma_2 + F_3 \cdot \sigma_3 + F_4 \cdot \sigma_4 + F_5 \cdot \sigma_5 + F_6 \cdot \sigma_6 + \\
&+ F_{11} \cdot \sigma_1^2 + 2F_{12} \cdot \sigma_1 \cdot \sigma_2 + 2F_{13} \cdot \sigma_1 \cdot \sigma_3 + 2F_{14} \cdot \sigma_1 \cdot \sigma_4 + 2F_{15} \cdot \sigma_1 \cdot \sigma_5 + 2F_{16} \cdot \sigma_1 \cdot \sigma_6 + \\
&+ F_{22} \cdot \sigma_2^2 + 2F_{23} \cdot \sigma_2 \cdot \sigma_3 + 2F_{24} \cdot \sigma_2 \cdot \sigma_4 + 2F_{25} \cdot \sigma_2 \cdot \sigma_5 + 2F_{26} \cdot \sigma_2 \cdot \sigma_6 + \\
&+ F_{33} \cdot \sigma_3^2 + 2F_{34} \cdot \sigma_3 \cdot \sigma_4 + 2F_{35} \cdot \sigma_3 \cdot \sigma_5 + 2F_{36} \cdot \sigma_3 \cdot \sigma_6 + \\
&+ F_{44} \cdot \sigma_4^2 + 2F_{45} \cdot \sigma_4 \cdot \sigma_5 + 2F_{46} \cdot \sigma_4 \cdot \sigma_6 + \\
&+ F_{55} \cdot \sigma_5^2 + 2F_{56} \cdot \sigma_5 \cdot \sigma_6 + \\
&+ F_{66} \cdot \sigma_6^2 = 1
\end{aligned} \tag{3}$$

For being invariant, the strength criterion is valid for all the coordinates systems.

Besides determining the criterion strength parameters, a stability condition must be respected for the criterion equation to represent a closed surface. On the plane stress state this condition would be: $F_{12} \leq \pm\sqrt{F_{11} \cdot F_{22}}$.

By applying the Tsai-Wu criterion for orthotropic materials, one can observe that the various strength parameters (F) get null, simplifying the use of the equation, which can be written in its expanded form as:

$$\begin{aligned}
&F_1 \cdot \sigma_1 + F_2 \cdot \sigma_2 + F_3 \cdot \sigma_3 + F_{11} \cdot \sigma_1^2 + F_{22} \cdot \sigma_2^2 + F_{33} \cdot \sigma_3^2 + 2F_{12} \cdot \sigma_1 \cdot \sigma_2 + 2F_{13} \cdot \sigma_1 \cdot \sigma_3 + \\
&+ 2 \cdot F_{23} \cdot \sigma_2 \cdot \sigma_3 + F_{44} \cdot \sigma_4^2 + F_{55} \cdot \sigma_5^2 + F_{66} \cdot \sigma_6^2 = 1
\end{aligned} \tag{4}$$

In the plane stress state, and considering orientations 1 and 2, eq. 4 can be reduced to:

$$F_1 \cdot \sigma_1 + F_2 \cdot \sigma_2 + F_{11} \cdot \sigma_1^2 + F_{22} \cdot \sigma_2^2 + 2F_{12} \cdot \sigma_1 \cdot \sigma_2 + F_{44} \cdot \sigma_4^2 = 1 \tag{5}$$

By applying the strength criterion for an orthotropic material, the number of parameters to be determined is reduced from 27 to 12.

For determining the strength parameters F_1, F_2, F_3, F_{11}, F_{22}, F_{33}, F_{44}, F_{55} and F_{66} it is necessary only uniaxial tensile, compression and shear tests.

Tab. 1 presents the strength coefficients F_i (i = 1, 2, 3) and F_{ii} (i = 1 to 6) of the polynomial tensor:

To determine the parameters F_{12}, F_{23} and F_{13}, it would be initially necessary to carry out biaxial tests, according, for example, to Suhling (1985) and Mascia et al. (2007). Due to the difficulties to carry out these tests, they can be replaced by a tensile, compression and shear as provided by Fig. 1, with the values shown in Tab. 2. It is possible to observe, however, that in cases 3 and 4 there are additional effects due to the tangential stresses.

Tab. 1: Coefficients F_i and F_{ii}.

$F_1 = \frac{1}{f_{t1}} - \frac{1}{f_{c1}}$	$F_2 = \frac{1}{f_{t2}} - \frac{1}{f_{c2}}$	$F_3 = \frac{1}{f_{t3}} - \frac{1}{f_{c3}}$
$F_{11} = \frac{1}{f_{t1} \cdot f_{c1}}$	$F_{22} = \frac{1}{f_{t2} \cdot f_{c2}}$	$F_{33} = \frac{1}{f_{t3} \cdot f_{c3}}$
$F_{44} = \frac{1}{f_{v4}^2}$	$F_{55} = \frac{1}{f_{v5}^2}$	$F_{66} = \frac{1}{f_{v6}^2}$

where: f_t - representing the tensile strength, f_c - compressive strength and f_v - shear strength.

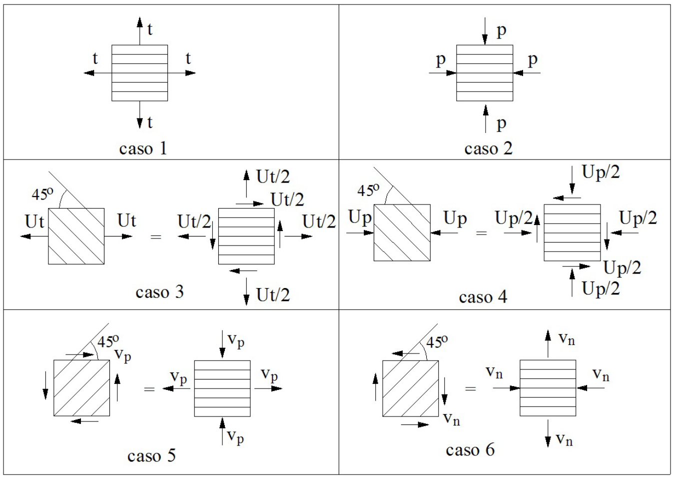

Fig. 1: – Biaxial and uniaxial tests (cases 1 to 6).

Tab. 2: Strength coefficient F_{12} and different stress combinations.

Case	Stress $(\sigma_1, \sigma_2, \sigma_{12})$	F_{12}
1	(t, t, 0)	$\frac{1}{2t^2} \cdot \left[1 - t \cdot (F_1 + F_2) - t^2 \cdot (F_{11} + F_{22})\right]$
2	(-p, -p, 0)	$\frac{1}{2p^2} \cdot \left[1 + p \cdot (F_1 + F_2) - p^2 \cdot (F_{11} + F_{22})\right]$
3	(Ut/2, Ut/2, Ut/2)	$\frac{2}{U_t^2} \cdot \left[1 - \frac{U_t}{2} \cdot (F_1 + F_2) - \frac{U_t^2}{4} \cdot (F_{11} + F_{22} + F_{44})\right]$
4	(-Up/2, -Up/2, -Up/2)	$\frac{2}{U_p^2} \cdot \left[1 + \frac{U_p}{2} \cdot (F_1 + F_2) - \frac{U_p^2}{4} \cdot (F_{11} + F_{22} + F_{44})\right]$
5	(vp, -vp, 0)	$-\frac{1}{2v_p^2} \cdot \left[1 - v_p \cdot (F_1 - F_2) - v_p^2 \cdot (F_{11} + F_{22})\right]$
6	(-vn, vn, 0)	$-\frac{1}{2v_n^2} \cdot \left[1 + v_n \cdot (F_1 - F_2) - v_n^2 \cdot (F_{11} + F_{22})\right]$

Other expressions for determining the interaction parameter F_{12} were developed by various other researchers and can be observed in Tab. 3.

Tsai-Wu Criterion and Hankinson's formula

Hankinson's formula can be deduced through a linear proximity of the polynomial tensor strength theory for orthotropic materials. The linear proximity of the strength theory has the following form:

$$F_1 \cdot \sigma_1 + F_2 \cdot \sigma_2 = 1 \tag{6}$$

Tab. 3: Determining the coefficient F_{12} according to some researchers.

Researcher	F_{12}
Ashkenazi (1965)	$\frac{1}{2}\cdot\left[\frac{4}{U_t^2}-\frac{1}{f_{t1}^2}-\frac{1}{f_{t2}^2}-\frac{1}{f_v^2}\right]$
Cowin (1979)	$\frac{1}{\sqrt{f_{t1}\cdot f_{c1}\cdot f_{t2}\cdot f_{c2}}}-\frac{1}{2\cdot f_v^2}$
Hoffman (1967)	$-\frac{1}{2\cdot f_{t1}\cdot f_{c1}}$
Liu (1984)	$\frac{1}{2}\cdot\left(\frac{1}{f_{t1}\cdot f_{c2}}+\frac{1}{f_{c1}\cdot f_{t2}}-\frac{1}{f_v^2}\right)$
Norris (1939)	$-\frac{1}{2\cdot f_{t1}\cdot f_{t2}}$
Tsai-Hill (1971)	$-\frac{1}{2\cdot f_t^2}$

The F_1 and F_2 coefficients should be determined in each of the four quadrants of the Cartesian plan of normal stress. In each quadrant, a different plan is determined and the resulting surface will be a parallelepipedon. If the orthotropic material is submitted to a tensile test in direction 1 and its strength is f_{t1}, provided all the stresses in Eq. 6 are zero except for σ_1,one can write that:

$$F_1 = \frac{1}{f_{t1}}$$

This value must be modified for each quadrant of the Cartesian plan of normal stress. Thus, for instance, in the quadrant where σ_1 is negative, the coefficient F_1 will be $-1/f_{c1}$. In the quadrant where all the normal stresses are compressive, Eq. 6 becomes:

$$\frac{\sigma_1}{f_{t1}}+\frac{\sigma_2}{f_{t2}}=1 \tag{7}$$

Consider now a tensile test scheme as the one in Fig. 2:

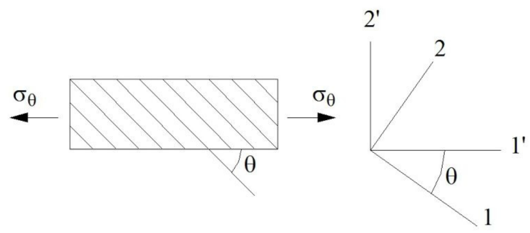

Fig. 2: Off-axis uniaxial tensile test.

In which the main stresses are worth:

$$\begin{aligned} \sigma_1 &= \sigma_\theta \cdot \cos^2\theta \\ \sigma_2 &= \sigma_\theta \cdot \sin^2\theta \\ \sigma_{12} &= \sigma_4 = \sigma_\theta \cdot \sin\theta \cdot \cos\theta \end{aligned} \tag{8}$$

By replacing the two first expressions in Eq. 7 we have Hankinson's formula is written by:

$$\sigma_\theta^- = \frac{f_{c1} \cdot f_{c2}}{f_{c1} \cdot \sin^2\theta + f_{c2} \cdot \cos^2\theta} \tag{9}$$

According to Cowin (1979), the fact that the linear proximity made as from the strength tensor leads to Hankinson's formula, associated to the fact that the experimental data in tests with wood are close to this formula should not be considered decisive to conclude that the linear proximity in the failure theory is sufficient for a material such as wood. This linear proximity presents several faults, being the first the fact that it does not take into account the shear or the interaction between the normal stresses.

Based on this, ignoring the linear terms of the failure criterion and making the necessary replacements of Eq. 8 in strength Eq. 5 for plane state of stresses with $\sigma_{12} = \sigma_4$, it has that:

$$\sigma_\theta^2 \cdot \left(F_{11} \cdot \cos^4\theta + F_{22} \cdot \sin^4\theta + (2F_{12} + \frac{1}{f_v^2}) \cdot \cos^2\theta \cdot \sin^2\theta \right) = 1 \tag{10}$$

According to if $F_{12} = \sqrt{F_{11} \cdot F_{22}} - \frac{1}{2 \cdot f_v^2}$, eq. 10 becomes:

$$\sigma_\theta = \pm \frac{\sqrt{f_{t1} \cdot f_{c1}} \cdot \sqrt{f_{t2} \cdot f_{c2}}}{\sqrt{f_{t2} \cdot f_{c2}} \cdot \cos^2\theta + \sqrt{f_{t1} \cdot f_{c1}} \cdot sen^2\theta} \tag{11}$$

Eq. 11 is similar to Hankinson's formula. However, the value adopted by Cowin for the interaction coefficient has the following value:

$$F_{12} = \sqrt{F_{11} \cdot F_{22}} - \frac{1}{2 \cdot f_v^2} = \sqrt{\frac{1}{f_{t1} \cdot f_{c1} \cdot f_{t2} \cdot f_{c2}}} - \frac{1}{2 \cdot f_v^2} \tag{12}$$

The coefficients F_{12} proposed by Hoffman and Cowin, in Tab. 3, do not demand the use of biaxial tests.

MATERIAL AND METHODS

There were tests in wood specimens for the determination of strengths that were used for the determination of the strength parameters used in Tsai-Wu criterion. The wood species used in this research was *Goupia glabra* species. The specimens for the tests were extracted in 6 beams with the following dimensions: 4.5 x 30 x 150 cm. The moisture content varied from 12 % to 14 %.

There were tensile tests in the parallel and perpendicular direction to the fibers, as well as in inclined directions in relation to the load of 15°, 30°, 45°, 60° and 75°. For each angle of the fibers 8 specimens were tested, reaching a total of 56 specimens.

Fig. 3 shows the specimen model adopted for this study for the tensile tests in parallel, perpendicular and inclined direction of the wood fiber in relation to the load. In Fig. 4, one can observe the equipment and the specimen for the tensile (Figs. 3, 4) test.

The load speed was 10 $MPa.min^{-1}$ according to NBR 7190 (1997). A universal tests machine was used with the capacity of 300 kN.

Compression tests were carried out in the parallel and perpendicular directions to the fibers, also as in the inclined direction in relation to the load of 15°, 30°, 45°, 60° and 75°. For each angle of the fibers, 12 specimens were tested, with a total of 84 specimens. A specimen of 4 x 4 x 12 cm was adopted. The load speed was 10 $MPa.min^{-1}$. A universal tests machine was used with the capacity of 300 kN. As in the tensile tests, only one load cycle was applied.

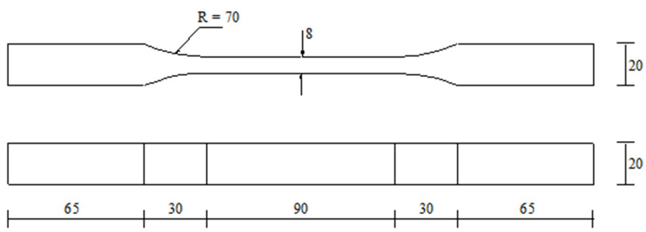

Fig. 3: Model of tensile specimen used in the tests (mm).

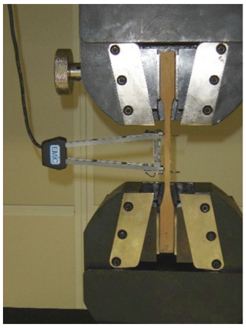

Fig. 4: Tensile test.

For the shear test there were 12 specimens and the load speed was 2.5 $MPa.min^{-1}$.

For the biaxial compressive tests it was necessary to develop a test equipment specific for the application of compressive force perpendicular to the fiber, as described by Nicolas (2006).

RESULTS

Through the results obtained from the tensile, compressive and shear tests in specimens it was possible to determine the strength values for the *Goupia glabra* wood species. In Tab. 4 presents such values.

Tab. 4: Strength values (MPa).

Angle (degrees)	Tensile strength	Compressive strength	Shear strength
0°	72.90	58.06	19.75
90°	4.65	20.09	--------

With the strengths results of Tab. 4, obtained in the tensile, compression and shear tests the strength parameters of the Tsai-Wu criterion were determined, as shown in Tab. 5.

The coefficient F_{12} in Tab. 5 is indicated with its maximum and minimum possible value; for higher values than the limits the failure surface becomes an open surface. The parallel orientation to the fibers of the wood was adopted as direction 1 and the perpendicular orientation to the fiber as direction 2.

Tab. 5 : Strength coefficients.

Parameter	Expression	Value
F_1	$\frac{1}{f_{t1}} - \frac{1}{f_{c1}}$	$-3.5061 \cdot 10^{-3} MPa^{-1}$
F_2	$\frac{1}{f_{t2}} - \frac{1}{f_{c2}}$	$1.6527 \cdot 10^{-1} MPa^{-1}$
F_{11}	$\frac{1}{f_{t1} \cdot f_{c1}}$	$2.3626 \cdot 10^{-4} MPa^{-2}$
F_{22}	$\frac{1}{f_{t1} \cdot f_{c1}}$	$1.0704 \cdot 10^{-2} MPa^{-2}$
F_{44}	$F_{44} = \frac{1}{f_{v4}^2}$	$2.5636 \cdot 10^{-3} MPa^{-2}$
F_{12}	$\pm\sqrt{F_{11} \cdot F_{22}}$	$\pm 1.5903 \cdot 10^{-3} MPa^{-2}$

By applying the strength parameters in the failure equation, eq. 5, for stress plane state, plane 1-2, it has that:

$$-\frac{1}{285.214} \cdot \sigma_1 + \frac{1}{6.037} \cdot \sigma_2 + \frac{1}{4232.574} \cdot \sigma_1^2 + \frac{1}{93.4185} \cdot \sigma_2^2 + \\ + 2F_{12} \cdot \sigma_1 \cdot \sigma_2 + \frac{1}{390.0625} \cdot \sigma_4^2 = 1 \tag{13}$$

By using Hankinson's formula, Eq. 1, for three different values of the exponent "n", it was possible to determine the strength curves based on the angle of the fibers in relation to the load. Fig. 5 presents a comparison between the results from the tensile tests and presented in Tab. 6, and the values obtained through Hankinson's formula (n = 1.5, n = 2, n = 2.5).

In Fig. 5 it is possible to evidence the large reduction of tensile strength obtained in the tests, due to the angle. For angles of up to 30° there were great differences between the estimates from Hankinson's formula, considering the various exponents. For angles above 30° there is no significant variation of strength. The values obtained in the tests were kept between the estimates from Hankinson's formula with n = 1.5 and n = 2.0.

Tab. 6: Tensile strength.

Angle (Degrees)	Tensile strength (MPa)
0	72.90
15	28.51
30	13.03
45	7.23
60	6.17
75	5.29
90	4.65

In Fig. 6 one can observe a comparison between the results obtained in the tensile tests, for various orientations of the wood fibers in relation to the load application, and the values obtained through Tsai-Wu strength criterion. The strength curves, from Tsai-Wu criterion, were estimated with the maximum and minimum values of the interaction coefficient F_{12}, and also for F_{12} null.

Fig. 7 shows the results obtained in the tensile tests and the strength estimates from Tsai-Wu and Hankinson which are closer to the laboratory results. Hankinson's formula, calculated with n = 1.5, and Tsai-Wu criterion, calculated with F_{12} = + limit, were the strength estimates that were closer to the values determined in the tests.

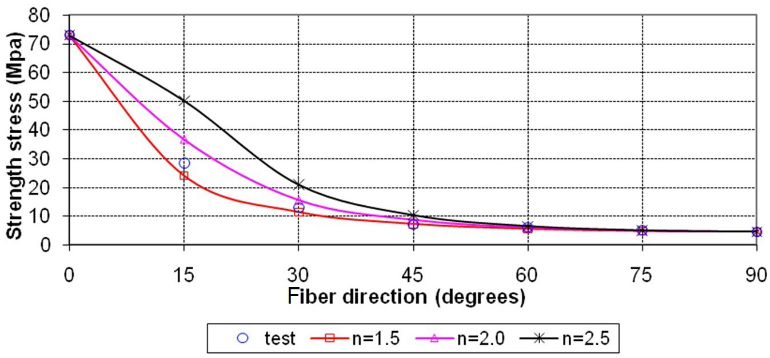

Fig. 5: Comparison between Hankinson's formula and data from tensile tests.

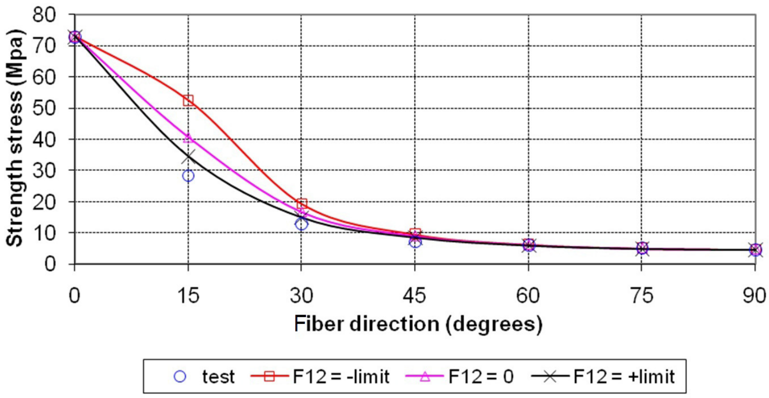

Fig. 6: Comparison between Tsai-Wu criterion and data from tensile tests.

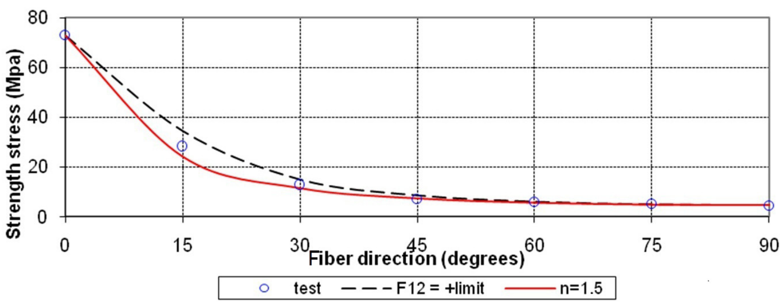

Fig. 7: Comparison between Hankinson's formula, Tsai-Wu criterion and data from tensile tests.

Statistical analysis

Using the statistic program Minitab, according to Ryan and Joiner (1994) and consonant to the statistic concepts of small samples, as presented by Box et al. (1978), the following statistic analysis of this study results are considered:

- μ_1 is the average of the 1st sample and μ_2 the average of the 2nd sample. In order to test if these two samples belong to the same universe, one can apply the following hypothesis: Ho : $\mu_1 = \mu_2$ versus H1 : $\mu_1 \neq \mu_2$. Calculating the significance through the expression:

$$t = (\overline{x}_1 - \overline{x}_2) / \sqrt{s_1^2 / n_1 + s_2^2 / n_2} \leq t_\varphi \ (P\%) \tag{14}$$

where: $\overline{x}_1$, $\overline{x}_2$ = estimates of the averages for the 1st and the 2nd samples; s_1 ,s_2 = standard deviations of the 1st and 2nd samples; n_1, n_2= number of elements of the 1st and 2nd samples; t_Φ (P %) = value of t' Student' with P % for reliability and (P %) = adopted reliability level.

Additionally, to check if the sample averages are statistically equivalent (if the interval of the difference between averages μ_2 and μ_1 has the zero), it is determined:

$$\begin{aligned} \Delta_- = (\overline{x}_1 - \overline{x}_2) - t^* \sqrt{s_1^2 / n_1 + s_2^2 / n_2} \leq \mu_2 - \mu_1 \leq (\overline{x}_1 - \overline{x}_2) + \\ t^* \sqrt{s_1^2 / n_1 + s_2^2 / n_2} = \Delta_+ \end{aligned} \tag{15}$$

In where: t^* is the value corresponding to P % of reliability. The expression of the freedom degrees (df) is equivalent to the following:

$$df = \frac{\left[(s_1^2 / n_1) + (s_2^2 / n_2) \right]^2}{\left[\dfrac{(s_1^2 / n_1)^2}{(n_1 - 1)} + \dfrac{(s_2^2 / n_2)^2}{(n_2 - 1)} \right]}$$

In the Minitab program, twosample test carries out these two assessments, simultaneously, as presented in Tabs. 7, 8.

Tab. 7: Estimates of tensile strength (MPa).

C1 Test	C2 n = 1.5	C3 n = 2.0	C4 n = 2.5	C5 Tsai (-limit)	C6 Tsai (0)	C7 Tsai (+limit)
			stress			
72.90	72.90	72.90	72.90	72.90	72.90	72.90
28.51	24.19	36.76	50.23	52.41	40.62	34.53
13.03	11.48	15.61	21.01	19.26	16.65	14.97
7.23	7.35	8.74	10.40	9.49	8.99	8.57
6.17	5.61	6.07	6.56	6.25	6.13	6.02
5.29	4.86	4.96	5.06	4.99	4.97	4.95
4.65	4.65	4.65	4.65	4.65	4.65	4.65

Based on the statistical analysis, it is concluded that there is no significant variation of the strength values coming from the tests, Hankinson's formula and Tsai-Wu criterion, for the uniaxial stress state despite the high variation of possible F_{12} values.

Tab. 8: Twosample test: $t_{\phi}(95\ \%) = 1.796$

Columns	Δ_-	Δ_+	$\lvert t^* \rvert$	Result
C1 x C2	-28.3	30.2	0.07	Ok!
C1 x C3	-31.3	27.9	0.13	Ok!
C1 x C4	-35.2	26.0	0.34	Ok!
C1 x C5	-35.4	26.0	0.33	Ok!
C1 x C6	-32.3	27.4	0.18	Ok!
C1 x C7	-30.8	28.2	0.09	Ok!

The values, obtained in the tests for tensile strength were below the estimates of the failure criterion. The closest results were obtained with F_{12} = + limit.

CONCLUSION

By considering the analysis of Tsai-Wu criterion, Hankinson's formula and together with the analysis of the data from the tests, the following conclusions can be presented:

- with the strength results obtained in the compressive, tensile and shear tests in specimens it was possible to estimate the failure curve, based on Tsai-Wu theory for the wood species analyzed in this study,

- despite the wide variation of the interaction coefficient F_{12} its value does not interfere in the estimates of tensile uniaxial strength. The same would not happen when analyzing a specimen in the stresses plane state,

- slight variation of the fibers angles in relation to the load generate large reductions in the strength value, revealing the anisotropic nature of wood,

- the data from the tests were within the estimates from Hankinson calculated with the exponent n = 1.5 and n = 2 and for the Tsai-Wu criterion the data obtained was calculated with the interaction coefficient F_{12} with its maximum positive value,

-comparing the test data, Hankinson's formula and Tsai-Wu criterion,one can conclude that the data were in an intermediary position between Hakinson's curves, with n = 1.5 and the Tsai-Wu strength curve with F_{12} = superior limit,

-the Tsai-Wu criterion, can be also used to estimate uniaxial compressive strength and strengths on the plane and tri-dimensional state of stresses,

- the main difficulty in the use of Tsai-Wu criterion is in determining the interaction coefficient Fij, as there are several expressions from various researchers that involve uniaxial or biaxial tests.

In addition, the biaxial compression tests, there is the need for future research work to carry out biaxial tensile, tensile-compression and compression-tensile tests so as to obtain a better evaluation of which coefficient F_{12} can fit to the test data more adequately.

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