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BENDING FAILURE MECHANICS AND BENDING STRENGTH PREDICTION MODELS OF FIBERBOARD BASED ON VERTICAL DENSITY PROFILE

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ABSTRACT

In order to develop the bending failure mechanics and establish the bending strength prediction models, fiberboards with uniform, "U" shape and "V" shape vertical density profiles (VDP) were manufactured, and their properties were evaluated and compared based on the elastic mechanics and laminated beam theory. The VDP of inertia (T_z) and the decision coefficient of bending failure types (K) were defined and used for building modulus of elasticity (*MOE*) and static bending intensity (*MOR*) prediction models. The results showed that the quantifying factors of fiberboard can be divided into panel thickness, maximum density, minimum density and T_z. The fracture failure occurred when K was bigger than 1, while shear failure occurred when K was smaller than 1. K was in connection with maximum density, minimum density, panel thickness and T_z. *MOE* of fiberboard was relevant to T_z and panel thickness. *MOR* of fiberboard had relation with maximum density, T_z and minimum density. The bending failure mechanics established by this study can explain and predict bending failure effectively and the bending strength prediction model also had a very strong predictive ability.

KEYWORDS: Fiberboard, vertical density profile (VDP), modulus of elasticity (*MOE*), static bending intensity (*MOR*), prediction model.

INTRODUCTION

Fiberboard is manufactured by series of production processes, such as fiber separation, fiber treatment, formation, hot pressing, etc., using plant fibers as its main raw material. Due to its great performance and low price, fiberboard is widely applied in the areas like construction, furniture design and vessel manufacturing, etc. It is usually used as wallboard, floor, and cement molding board, ceiling, hollow door, furniture board and so on. Fiberboard usually needs higher intensity when it comes to building material, which often achieved by increasing its average density and the quantity of resin usage, both of which cannot necessarily improve board intensity in the practice.

Vertical Density Profile (VDP), namely, the density variance of wooden composite on thickness direction, which is the important structure feature and physical property of fiberboard, and a key factor which shows great impacts on physical and mechanical performances of wood-based panels (Kelly 1977, Xu 1999). Traditional VDP usually has geometric symmetry shape along thickness axis, higher density in surface, lower in sandwich layer, the highest density appears at a certain thickness position from panel, which is called "M" type (M-fiberboard). Changing hot pressing process can produce different shape of VDP. Divided by the shape of VDP, fiberboard can be divided into homogeneous fiberboards (H-fiberboard), M-fiberboard, fiberboards with "V" shape VDP (V-fiberboard) and fiberboards with "U" shape VDP (U-fiberboard).

Wood fiber is poor conductor for heat, in process of hot-pressing, the solidification of resin, from surface to sandwich layer, happens gradually and continuously, as mat temperature achieving resin curing temperature from outside to inside. The final density of different position on thickness direction depends on pressure of resin solidification and inner environment of mat during hot-pressing (Wang et al. 2000, Zhang 2009a). Therefore, during the process of fiberboard's hot-pressing, temperature gradient, moisture gradient and variance of resin solidifying level of inner mat prompt uneven distribution of VDP. Pre-curing layer can be eliminated by water spraying on surface effectively, section closure can reduce transition zone thickness and increase sandwich lay density (Dai 2004, Zhang 2009b).

As the most important mechanical index of fiberboard, many researches of bending performance only focus on effect of traditional VDP on modulus of elasticity (*MOE*) and static bending strength (*MOR*), on which amount of prediction models have been established (Carll and Link 1988, Suo and Bowyer 1995, Steidl et al. 2003, Painter et al. 2006, Zhang 2009, Jin 2009). In the paper, through investigating the effect of VDP on bending strength for fiberboards with three shapes of VDP and analyzing their bending failure mechanism, more accurate prediction models of elasticity and static bending strength were established based on theory of elastic mechanics, laminated beam theory and mechanics of materials.

MATERIAL AND METHODS

Raw materials

The fiber, Phenol-formaldehyde liquid resin and emulsified wax used in the studies were from a fiberboard mill located in Beijing which primarily using a mixture of hardwood fiber.

The solid loading levels of resin and wax were 3.5 % and 0.5 % based on the oven-dry weight of the fiberboard respectively. According to the moisture content of each batch of dry fibers, water was added to keep the target moisture content for each mat.

Board fabrication

Fiberboards with different VDP shape were fabricated by changing two selected variables. Board density was selected as one variable. The other variable was board structure (shape of VDP). The thickness gauge was adopted to ensure target thickness of each fiberboard at 13±0.5 mm during hot-pressing. The pressure for hot-pressing and the moisture content of each mat were shown in Tab. 1. All fiberboards were sanded to achieve 12 mm target thickness.

Homogeneous fiberboard (H-fiberboard)

The nominal density levels of H–fiberboard ranged from 500 to 1000 $kg.m^{-3}$ at an interval of 100 kg.m^{-3}, resulting in six target density levels.

Tab. 1: Conditions of hot pressing for fiberboard.

Hot pressing temperature (°C)	Rise time (s)	High pressure (MPa)	Holding time at high-pressure (s)	Low-pressure (MPa)	Holding time at low-pressure (s)	Moisture content (%)
180	20	3	90	1	330	8

To produce fiberboards with homogeneous VDP through the board thickness, a special pre-determined warm pre-pressing cycle, which was similar to the previously reported "cold-pressing" method (Geimer et al. 1975, Wang et al. 2000, Jin 2009), was employed. All mats were pre-pressed to the target thickness of 13 mm at a platen temperature of 70 ± 1°C, and then held at the target thickness until the core temperature reached the platen temperature. Subsequently, both the top and bottom platens were heated to 180°C. The boards were removed as soon as the core temperature reached 120°C. The whole pressing cycle ranged from 20 to 25 min depending on the board density, with denser boards requiring longer time.

Fiberboard with "V" shape VDP (V-fiberboard)

The nominal density levels of V–fiberboard were ranged from 600 to 900 kg.m^{-3}. To produce fiberboards with "V" shape VDP, mats were under humidifying treatment on both of their surfaces firstly, and then removed to hot presser under the conditions shown in Tab. 1.

Fiberboard with "U" shape VDP (U-fiberboard)

Fiberboards with target density levels varying from 600 to 900 kg.m^{-3} were made by the method as follow. To produce fiberboards with "U" shape VDP, all mats were pre-pressed to 16 mm in thickness at room temperature firstly, then humidifying treatment was applied to both surfaces of mats. Subsequently, mats were moved to hot presser under the pressing condition as V-fiberboard.

Specimen preparation and Testing

The pressed fiberboards were conditioned for one week in a standard conditioning climate of 25°C and 60 % relative humidity before being cut into test specimens. For each board, 50 × 50 mm specimens were prepared for VDP determination, 75 × 317 mm specimens were cut for bending tests. Prior to testing, all the specimens were further conditioned under the same standard conditioning climate until they were equilibrated.

CreCon's X-ray densitometer was used to measure the VDP of fiberboard specimens. The tests for other properties were conducted according to the standard GB/T 17657—1999.

Theoretical basis

The bending mechanical properties (*MOE*, *MOR*) of fiberboards were tested by the 3-point bending loading method. The top surface of panel was compression area but the bottom surface was tension area. The sandwich layer was shearing area. The elastic mechanics and laminated beam theory which were adopted in the paper were under the conditions as follow:

- The bonded connectivity between adjacent fibers in fiberboard, namely, continuity hypothesis;
- Tiny load speed of bend loading, panel keeps in balanced state under each loading, namely, pure bending hypothesis;

- To be divided into countless sheets along thickness direction via differential calculus, every single sheet mechanical property depends on density:

$$MOE = f(D) \tag{1}$$

$$\sigma_{max} = F(D) \tag{2}$$

$$\tau_{max} = G(D) \tag{3}$$

where: σ_{max} is bending tensile strength (MPa)
τ_{max} is bending shear strength (MPa).

- Before or after bending deformation, each sheet parallels neutral panel, meanwhile length modification is tiny;
- Other factors are equal, e.g., wood fiber, glue, resin usage and panel moisture, etc.

Characteristic and quantized model of VDP

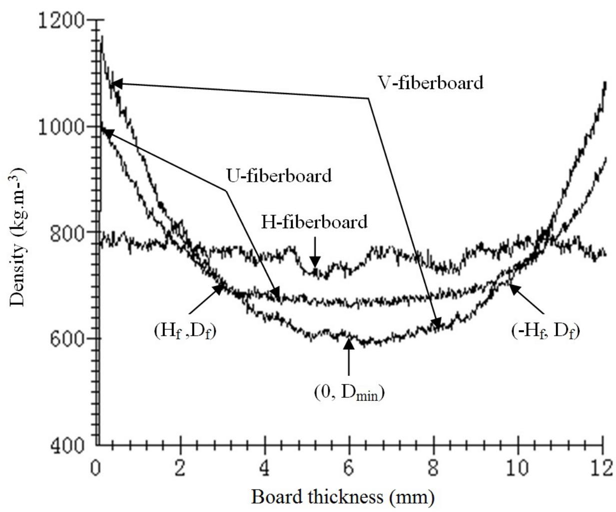

Fig. 1: VDPs of H-fiberboard, V-fiberboard and U-fiberboard with 500 kg.m^{-3} of average density.

Fig. 1 displays the VDP of H-fiberboard, U-fiberboard and V-fiberboard with the average density of nearly 800 kg.m^{-3}. It is shown that there was no pre-curing layer existed in H-fiberboard, U-fiberboard and V-fiberboard. VDP of H-fiberboard was uniform in the main, whereas VDP of U-fiberboard and V-fiberboard was inhomogeneous and the appeared characteristics as follow: Firstly, maximum density was appeared at panel surface and minimum density existed in sandwich layer; Secondly, panel was divided into top surface layer, sandwich layer and bottom surface layer by two inflection points [($-H_f$, D_f) and (H_f, D_f)], which existed in both V-fiberboard and U-fiberboard, however VDP of V-fiberboard was steeper than U-fiberboard; Lastly, VDP was exactly symmetrical around the axis. Consequently, the highest density appeared at coordinates ($0.5H$, D_{max}) and ($-0.5w$, D_{max}). The coordinates of the lowest density of mat, which was the middle position of mat, was at (0, D_{min}).

VDP of H-fiberboard can be described as:

$$D(h) = D_A \tag{4}$$

VDP of U-fiberboard can be divided into top surface layer, sandwich layer and bottom surface layer. VDP of sandwich layer was uniform and D_f was equal to D_{min}. In consequence, the relationship between thickness and the density of top surface layer and bottom surface layer can be described as quadratic function:

$$D(h) = \begin{cases} ah^2 + bh + c & -0.5H \le h \le -H_f \\ D_{min} & -H_f < h < H_f \\ ah^2 - bh + c & H_f \le h \le 0.5H \end{cases} \tag{5}$$

where: *a, b, c* is the undetermined coefficients of quadratic function separately;
h is the thickness of different position (mm)
H is the panel thickness (mm)

In terms of dots on quadratic parabola of $(0.5H, D_{max})$ and (H_f, D_{min}), the solution formula of *a, b, c* can be concluded:

$$\begin{cases} a(0.5H)^2 - 0.5bH + c = D_{max} \\ a(H_f)^2 - bH_f + c = D_{min} \\ D_{min}H_f + \int_{H_f}^{0.5H} \left[ah^2 - bh + c\right]dh = 0.5HD_A \end{cases} \tag{6}$$

VDP of H-fiberboard can be divided into top surface, sandwich layer and bottom surface. By means of setting D_f equal to D_A, the relationship between thickness and the density of top surface layer and bottom surface layer can be described as linear function, but the relationship between the density of sandwich layer and thickness can be described as quadratic function:

$$D(h) = \begin{cases} -mh + n & -0.5H \le h \le -H_f \\ ah^2 + bh + D_{min} & -H_f \le h \le 0 \\ ah^2 - bh + D_{min} & 0 \le h \le H_f \\ mh + n & H_f \le h \le 0.5H \end{cases} \tag{7}$$

where: m, n was undetermined coefficients of linear function separately.

In terms of dots on quadratic parabola of $(0.5H, D_{max})$ and $(0, D_{min})$, the solution formula of *a, b, c, m, n* can be concluded.

$$\begin{cases} 0.5Hm + n = D_{max} \\ H_f m + n = D_A \\ a(H_f)^2 - bH_f + c = D_A \\ c = D_{min} \\ \int_0^{H_f} (mh + n)dh + \int_{H_f}^{0.5H} \left[ah^2 - bh + c\right]dh = 0.5HD_A \end{cases} \tag{8}$$

The impact model of VDP on tensile stress

As the density along thickness direction is different for H-fiberboard, modulus of elasticity is different either (Dai 2008, Zhang 2009b). The result of modulus of elasticity coming from above formula is the one for whole panel, which also has positive correaltion of gradient with the relation curve of bend loading and defelction.

In linear area of the relation of bend loading and deflection, any point along the direction of cross section of fiberboard submits Hooks' law (Jin 2009, Zhang 2009b):

$$\sigma(h) = MOE\frac{h}{\rho} = f\left[D(h)\right]\frac{h}{\rho} \tag{9}$$

where: ρ is radius of curvature of middler layer in bending process (mm)
σ(h) is tensile stress of different thickness of fiberboard (MPa).

In terms of statics, the sum of normal strees of each point on panel cross section is 0, the sum of force moment of each point eqauls to the external one, equations are (Zhang 2009b, Jin 2009):

$$\int_A \sigma \mathrm{dA} = B\int_{-0.5H}^{0.5H}\sigma \mathrm{dh} = 0 \tag{10}$$

$$\int_A \sigma h \mathrm{dA} = B\int_{-0.5H}^{0.5H}\sigma h \mathrm{d}h = M = 0.5PL \tag{11}$$

where: M is the moment of couple (N.mm)
B is the panel width (mm)
P is the pressure of loadig (N)
L is span length between two bearings (mm).
Substituting equation (9) into (11), then,

$$M = 0.5PL = \frac{J_z}{\rho} \tag{12}$$

where: $J_z = B\int_{-0.5H}^{0.5H} f\left[D(h)\right]h^2 \mathrm{d}h$, can be defined as the elastic modulus of inertia, (GPa.mm^4).

Substituting equation (12) into (9), then,

$$\sigma(h) = \frac{PL}{2}\frac{f\left[D(h)\right]h}{J_z} \tag{13}$$

Bending failure mechanics

The horizontal compressive strength was far bigger than tensile strength and VDP unusually was non-uniform, which was the most important physical properties to fiberboard, thereby the bottom surface holding the biggest tensile stress and sandwich layer appearing the least shear strength were the dangerous zones where bending failure first may happen. Along with increasing of the loading pressure, fracture failure occurred when the tensile stress of bottom surface reached tensile strength was earlier than the shear stress of sandwich layer reached shear strength, otherwise shear failure occurred.

The bending tensile strength and shear strength can be obtained from equation (2) and (3):

$$\sigma_{\max} = F(D_{\max}) \tag{14}$$

$$\tau_{\max} = G(D_{\min}) \tag{15}$$

The bending shear stress in sandwich layer of fiberboard can be calculated based on mechanics of materials:

$$\tau = 1.5\frac{P}{BH} \tag{16}$$

The fracture failure and shear failure were two fundamental types of bending failure, and the coefficient of determination for types of bending failure can be obtained from the equation as follows:

$$K = \frac{\tau_{\max}}{\tau}\frac{\sigma}{\sigma_{\max}} = \frac{G(D_{\min})f\left[D_{\max}\right]BLH^2}{6F(D_{\max})J_z} \tag{17}$$

where: K is the decision coefficient of bending failure types.

The equation (17) demonstrated that the types of bending failure was not only in connection with maximum density and minimum density, but also had more to do with the elastic modulus of inertia. The fracture failure occurred when K was bigger than 1 and shear failure occurred when K was smaller than 1.

Static bending strength (MOR) prediction model

MOR means bending intensity of penal, the formula is (Ding 2009, Zhang 2009a):

$$MOR_{\mathrm{T}} = \frac{3L}{2B \cdot H^2}P_{\max} \tag{18}$$

where: P_{max} is the pressure of loading when panel occurred bending failure, N.

According to the equation (18), *MOR* was determined by P_{max} which obtained only by bending failure test. Based on bending failure mechanics as mentioned above, *MOR* prediction model can be established as follows:

The fracture failure occurred when K was bigger than 1.

The tensile strength can be calculated from equation (13):

$$\sigma_{\max} = F(D_{\max}) = \frac{P_{\max}L}{2B}\frac{f\left[D_{\max}\right]0.5H}{\int_{-0.5\mathrm{H}}^{0.5\mathrm{H}} f[D(h)]h^2\,\mathrm{d}h} \tag{19}$$

The *MOR* of fiberboard when bending fracture failure occurred can be found form equation (18) and (19):

$$MOR_{\mathrm{pf}} = \frac{3F\left(D_{\max}\right)\int_{-0.5\mathrm{H}}^{0.5H} f[D(h)]h^2\,\mathrm{d}h}{f\left[D_{\max}\right]H^3} = \frac{J_z F\left(D_{\max}\right)}{2I_z f\left(D_{\max}\right)} \tag{20}$$

where: $I_z = \frac{BH^3}{12}$ is the moment of inertia of the entire board, (mm^4).

The bending shear failure occurred when K was smaller than 1.

The shear strength can be calculated from equation (15) and (16):

$$\tau_{max} = G(D_{min}) = 1.5\frac{P_{max}}{BH} \tag{21}$$

The *MOR* of fiberboard when bending shear failure occurred can be obtained from equation (18) and (21):

$$MOR_{PS} = \frac{G(D_{min})L}{H} \tag{22}$$

Modulus of elasticity (*MOE*) prediction model

MOE of fiberboard is the gradient of linear relation area between bend deflection and loading in process of three point load bending (Zhang 2009a, Ding 2009). The formula is:

$$MOE_T = \frac{L^3}{4BH^3}\frac{\Delta P}{\Delta V} = \frac{L^3}{4BH^3}\frac{P_e}{V_e} \tag{23}$$

where: ΔP is linear load differenc under proportional limit (N)

ΔV is linear deflection difference, under proportional limit (mm).

To hypothesize equivalent bending modulus of elasticity of penal for MOE_P, the equation (24) can be obtained based on mechanics of materials:

$$MOE_P = \frac{M\rho}{I_z} = \frac{0.5PL\rho}{I_z} \tag{24}$$

Substituting equation (12) into (24), then:

$$MOE_P = \frac{B\int_{-0.5H}^{0.5H} f\left[D(h)\right]h^2 \mathrm{d}h}{I_z} = \frac{J_z}{I_z} \tag{25}$$

The equation (25) showed that *MOR* of fiberboard was in direct proportion to the elastic modulus of inertia (J_z) and in inverse proportion to moment of inertia (I_z).

RESULTS

Tab. 2 illustrates that the density variation (F_d) of VDP with H-fiberboard was below 10 % and the density variation (F_d) of VDP with H-fiberboard was between 50 % and 70 %, but the density variation (F_d) of VDP with V-fiberboard was more than 70 %. In the V-fiberboard occurred bending shear failure, but in U-fiberboard and H-fiberboard occurred bending fracture failure. Therefore, the bending shear failure occurred when density variation of VDP was bigger than 70 % and bending fracture failure occurred when density variation of VDP was lower than 70 %.

Fig. 2 shows that under the same average density of the three types of fiberboard, *MOR* of U-fiberboard was biggest, followed by V-fiberboard and H-fiberboard consequently. Each type of fiberboard had a linear regression equation between *MOR* and average density.

Fig. 3 indicates that U-fiberboard and H-fiberboard had an obvious linear relationship between MOR and maximum density but V-fiberboard had no obvious relationship between MOR and maximum density. Under the same maximum density of three types of fiberboard, MOR of H-fiberboard was biggest, followed by U-fiberboard and V-fiberboard sequentially.

Tab. 2: VDP quantization factors of different fiberboard.

Panel	H (mm)	D_A (kg.m^{-3})	D_{max} (kg.m^{-3})	D_{min} (kg.m^{-3})	F_d	*The type of failure*	H_f *(mm)*	*a*	*b*	*c*	*m*	*n*
H-1	12	509	531	482	0.10	fracture	/	/	/	/	/	/
H-2	12	608	643	589	0.09	fracture	/	/	/	/	/	/
H-3	12	701	733	687	0.07	fracture	/	/	/	/	/	/
H-4	12	789	825	745	0.10	fracture	/	/	/	/	/	/
H-5	12	893	919	864	0.06	fracture	/	/	/	/	/	/
H-6	12	989	1027	958	0.07	fracture	/	/	/	/	/	/
V-1	12	638	904	481	0.93	shear	3.12	11.52	-14.37	481	92	350
V-2	12	655	978	515	0.71	shear	2.92	9.31	-20.75	515	105	349
V-3	12	688	1001	520	0.70	shear	2.97	8.55	-31.16	520	103	381
V-4	12	701	1126	544	0.83	shear	2.82	10.01	-27.46	544	134	324
V-5	12	729	1169	548	0.85	shear	2.88	11.56	-29.57	548	141	323
V-6	12	760	1173	549	0.72	shear	2.7	1.15	-75.05	549	125	422
V-7	12	842	1288	599	0.74	shear	2.78	3.21	-78.48	599	139	457
V-8	12	830	1245	586	0.83	shear	2.86	4.91	-71.27	586	132	452
V-9	12	904	1389	598	0.72	shear	3	11.93	-66.20	598	162	419
U-1	12	516	799	508	0.56	fracture	4.66	25.33	52.84	204	/	/
U-2	12	580	827	524	0.52	fracture	4	1.93	-132.16	-36	/	/
U-3	12	609	878	529	0.57	fracture	4	1.50	-159.50	-133	/	/
U-4	12	700	1029	603	0.61	fracture	3.88	2.16	-179.57	-126	/	/
U-5	12	698	1006	641	0.52	fracture	4.2	7.45	-126.83	-23	/	/
U-6	12	709	1104	633	0.66	fracture	4.7	29.18	-50.08	-247	/	/
U-7	12	745	1142	628	0.69	fracture	4	8.37	-173.32	-199	/	/
U-8	12	790	1195	661	0.68	fracture	4	6.59	-201.08	-249	/	/
U-9	12	833	1242	712	0.64	fracture	3.58	0.09	-218.17	-70	/	

Where: $F_d=(D_{max} - D_{min})/D_A$

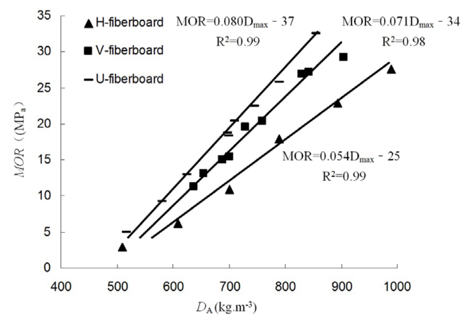

Fig. 2: Relationship between average density and MOR.

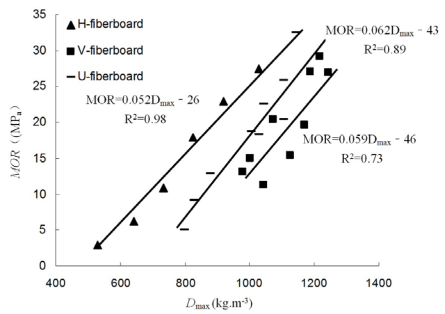

Fig. 3: Relationship between maximum density and MOR.

Fig. 4 illustrates that each type of fiberboard had a linear regression equation between *MOE* and average density. Under the same average density, *MOE* of the V-fiberboard was the biggest, followed by U-fiberboard and H-fiberboard consequently.

Fig. 5 indicates that *MOE* of fiberboard was not always raised only by improving maximum density.

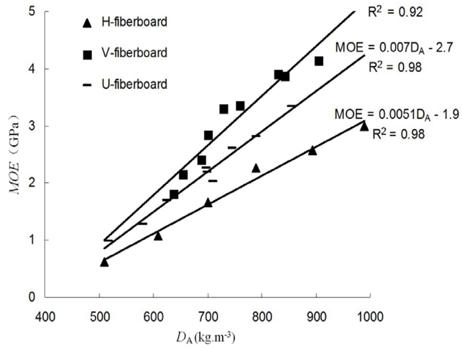

Fig. 4: Relationship between average density and MOE.

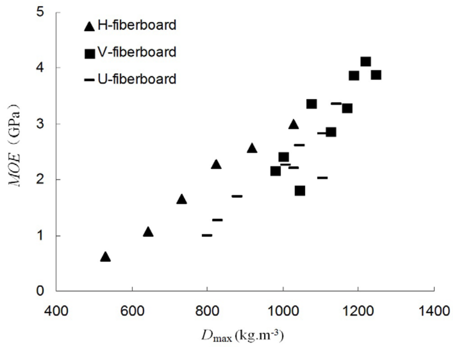

Fig. 5: Relationship between maximum density and MOE.

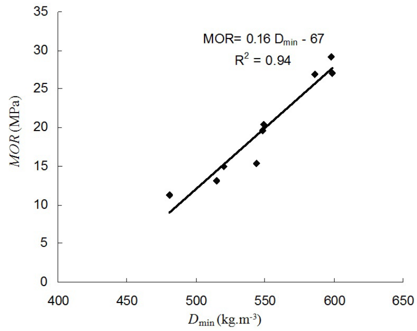

Fig. 6: Relationship between minimum density of V-fiberboard and MOR.

Fig. 6 shows that there was a linear equation between *MOR* and minimum density of V-fiberboard:

$$MOR = 0.16 D_{\min} - 67 \tag{26}$$

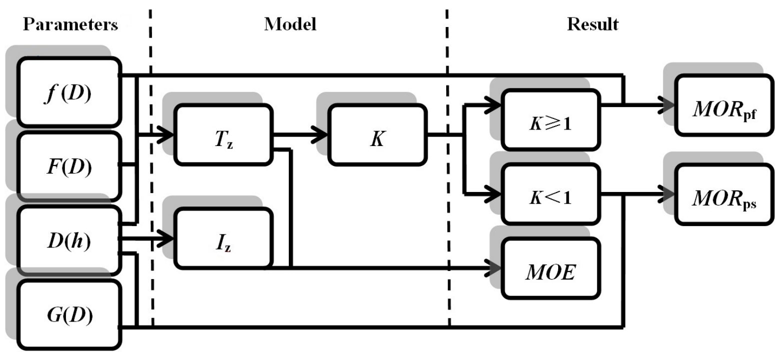

Fig. 7: Theoretical route of bending properties prediction model.

Fig. 7 illustrates the theoretical model of bending properties based on VDP characteristic of fiberboard. When VDP, *f (D), F (D)* and *G (D)* were fixed, the VDP of inertia (T_z) and the decision coefficient of bending failure types (K) can be determined, *MOE*, types of bending failure and *MOR* of fiberboard can be predicted consequently.

Tab. 3: Comparison of calculated model value and actual value.

Panel	K	MOE_P (MPa)	MOE_T (MPa)	R_{MOE}	MOR_P (MPa)	MOR_T (MPa)	R_{MOR}
V-1	0.54	2.02	1.80	0.12	10.29	11.25	-0.09
V-2	0.71	2.30	2.15	0.07	14.73	13.13	0.11
V-3	0.70	2.43	2.41	0.01	16.53	15.00	0.10
V-4	0.73	2.84	2.85	0.01	20.37	18.42	0.11
V-5	0.70	3.01	3.29	-0.09	21.01	19.58	0.07
V-6	0.68	3.14	3.36	-0.07	21.17	20.42	0.04
V-7	0.80	3.63	3.86	-0.06	29.17	27.08	0.08
V-8	0.78	3.45	3.89	-0.11	27.09	26.88	0.01
V-9	0.72	3.97	4.13	-0.04	29.01	29.17	-0.01
U-1	1.46	1.10	0.99	0.11	6.58	6.10	0.08
U-2	1.34	1.38	1.27	0.09	10.23	9.17	0.33
U-3	1.27	1.50	1.70	-0.12	13.55	12.92	0.05
U-4	1.48	2.07	2.20	-0.06	19.43	18.33	0.06
U-5	1.82	2.04	2.26	-0.10	19.01	18.75	0.01
U-6	1.78	1.98	2.03	-0.02	18.79	20.42	-0.08
U-7	1.47	2.33	2.60	-0.11	22.20	22.50	-0.01
U-8	1.55	2.54	2.81	-0.10	24.40	25.83	-0.06
U-9	1.59	2.97	3.34	-0.11	28.70	32.50	-0.12

Where: R_{MOE}=(MOE_P -MOE_T)/ MOE_T; R_{MOR}=(MOR_P -MOR_T)/ MOR_T

Tab. 3 shows that the decision coefficient of bending failure types (K) of V-fiberboard was between 0.54 and 0.8, and the decision coefficient of bending failure types (K) of U-fiberboard was between 1.27 and 1.82, which was in accordance with expected value. The differential value (R) of calculated value and actual value of *MOR* and *MOE* was less than 0.14. As a result, the prediction models of *MOE* and *MOR* established by this study had a very strong predictive ability.

DISCUSSION

In view of uniform density profile of H-fiberboard, the elastic modulus of inertia can be obtained by equation (27):

$$J_z = B\int_{-0.5H}^{0.5H} f\left[D(h)\right]h^2 \mathrm{d}h = f\left(D_A\right)I_z \tag{27}$$

MOE of H-fiberboard can be described by substituting equation (27) into (26):

$$MOE_P = \frac{J_z}{I_z} = f(D) \tag{28}$$

Combined with the linear regression equation between *MOE* and average density of H-fiberboard shown in Fig. 4 and equation (28), $f(D)$ can be obtained as follows:

$$f(D) = 0.0051D - 1.9 \tag{29}$$

MOR of H-fiberboard can be established by equation (20):

$$MOR = \frac{3F(D_{\max})\int_{-0.5H}^{0.5H} f[D(h)]h^2 \mathrm{d}h}{0.5f[D_{\max}]H^3} = \frac{J_z F(D_{\max})}{2I_z f(D_{\max})} = \frac{F(D)}{2} \tag{30}$$

Combined with the linear regression equation between *MOR* and average density of H-fiberboard shown in Fig. 2 and equation (30), $F(D)$ can be obtained as follows:

$$F(D) = 0.108D - 50 \tag{31}$$

The relation between $G(D)$ and *MOR* can be derived from equation (22) when H was 12 mm and L was 200 mm:

$$G(D_{\min}) = 0.06MOR \tag{32}$$

Combined equation (26) with equation (32), $G(D)$ can be described as follows:

$$G(D) = 0.025D - 4 \tag{33}$$

According to comments mentioned above, the linear relationships of mechanical property and density ($f(D)$, $F(D)$, $G(D)$) were depended on material property of fiberboard.

Combined equation (20) and equation (29), the elastic modulus of inertia can be acquired as follows:

$$J_z = B\int_{-0.5H}^{0.5H} f\left[D(h)\right]h^2 \mathrm{d}h = 0.0051B\int_{-0.5H}^{0.5H} D(h)\,h^2 \mathrm{d}h - 1.9I_z \tag{34}$$

where: $T_z = \int_{-0.5H}^{0.5H} D(h)\,h^2 \mathrm{d}h$. T_z can be defined as the VDP of inertia which described the effect of VDP on bending properties.

Combined with equation (17), (20), (25) and (34), the bending properties theoretical model can be shown as equation (35), (36) and (37):

$$J_z = 0.0051BT_z - 1.9I_z \tag{35}$$

$$K = \frac{\tau_{\max}}{\tau}\frac{\sigma}{\sigma_{\max}} = \frac{(0.025D_{\min} - 4)(0.0051D_{\max} - 1.9)LH^2}{(0.108D_{\max} - 50)(0.0306T_z - 0.95H^3)} \tag{36}$$

$$MOR_{\mathrm{Pf}} = \left(0.0612\frac{T_z}{H^3} - 1.9\right)\frac{(0.108D_{\max} - 50)}{(0.0102D_{\max} - 3.8)} \tag{37}$$

$$MOR_{\mathrm{PS}} = 0.16D_{\min} - 67 \tag{38}$$

$$MOE_{\mathrm{P}} = \frac{J_z}{I_z} = \frac{0.0051BT_z}{I_z} - 1.9 \tag{39}$$

Combined with equation (5) to equation (7) and equation (35) to equation (39), the *MOR* and *MOE* of any fiberboard with V-fiberboard, U-fiberboard and H-fiberboard can be predicted.

In terms of the results mentioned above, *MOR* can be improved by the raising density variation befittingly, however it would drop when the density variation of VDP was too large to let bending shear failure occur, so *MOR* of fiberboard was not always raised only by improving maximum density or average density. When the decision coefficient of bending failure types (*K*) was bigger than 1, the bending fractured failure occurred. The equation (37) shows that *MOR* was connected with maximum density and the VDP of inertia (T_z) which was determined by whole VDP shape of fiberboard. When the decision coefficient of bending failure types (*K*) was smaller than 1, the bending shear failure occurred at the zone of minimum density and there was a linear equation of *MOR* and minimum density (equation 38). So *MOR* of fiberboard had relation with maximum density, T_z and minimum density.

In the same way, *MOE* of fiberboard was not only raised by improving maximum density all the time. The equation (37) shows that *MOE* of fiberboard was only connected with the VDP of inertia (T_z) and the moment of inertia (I_z). Therefore, there was no direct relationship between maximum density and *MOR*. Under the same average density of three types of fiberboard, the moment of inertia (I_z) of V-fiberboard is the biggest, followed by U-fiberboard and H-fiberboard in turn, so *MOE* of V-fiberboard is the biggest, followed by U-fiberboard and H-fiberboard in turn.

CONCLUSION

Within the materials, parameters, and methods examined in this study, the main conclusions can be drawn as follows:

- Under the same average density of three types of fiberboard, *MOR* of the U-fiberboard is the biggest, followed by V-fiberboard and H-fiberboard consequently; but when it comes to *MOE*, V-fiberboard is the biggest, followed by U-fiberboard and H-fiberboard in turn.
- Raising the density variation of VDP befittingly can improve *MOR*, but *MOR* of fiberboard will drop when the density variation of VDP was too large to generate bending shear failure. *MOR* of fiberboard is not always raised only by improving maximum density or average density. The quantifying factors of fiberboard can be divided into panel thickness, maximum density, minimum density and VDP of inertia (T_z).
- The fracture failure occurs when the decision coefficient of bending failure types (*K*) is greater than 1 and shear failure occur when K is lower than 1. K is in connection with maximum density, minimum density, panel thickness and the elastic modulus of inertia.
- Based on the bending strength prediction model established by this study, *MOE* of fiberboard is relevant to T_z and panel thickness; *MOR* of fiberboard has relation to maximum density and T_z or minimum density depended on the decision coefficient of bending failure types.
- The bending failure mechanics established by this study can explain and predict bending failure effectively and the bending strength prediction model also has a very strong predictive ability.

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