DETERMINING THE PARAMETERS OF WOOD MACHINABILITY AS A FUNCTION OF TANGENTIAL CUTTING FORCE DURING THE PROCESS OF MACHINING WOOD BY ROUTING

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ABSTRACT

Machinability has been defined, in a general way, as an ability of a certain structural material to be machined by cutting or deformation through common, economical production techniques and technologies. However, a significant disadvantage of this definition is that it does not define the way in which this ability (machinability) is quantified and measured. This research deals with ways of determining numerical values of parameters of machinability during the process of machining oak wood by longitudinal routing. In order to determine the factors of machinability in the function of tangential cutting force the authors used multifactor orthogonal linear plan of experiment. Analyzing the position of straight lines in the given diagrams and the values of the exponents in the model the following can be concluded: Average tangential cutting force increases with the increase of any of the analyzed factors; With regard to intensity, cutting depth has a greater effect on the cutting force; If the cutting depth increases, the increase of cutting force is faster when feed per edge is greater. If the feed per edge is increased, the increase of tangential cutting force is more significant when the value of cutting depth is greater.

KEYWORDS: Cutting force, machinability, oak wood, regime, routing.

INTRODUCTION

According to Westkamer and Licher (1989, 1990), and Skakić (1992) machinability is an important term in the theory of machining materials by cutting. In the field of wood machining the problem of machinability hasn’t been thoroughly explored; it has only been described indirectly by variety of parameters, such as: Hardness, toughness and plasticity of wood, by elements of machining regime etc.
Machinability has not been precisely defined as a scientific term. Most often it has been defined, in a general way, as an ability of a certain structural material to be machined by cutting or deformation through common, economical production techniques and technologies (machines, tools, methods).

However, a significant disadvantage of this definition is that it does not define the way in which this ability (machinability) is quantified and measured.

Since it is, for now, still impossible to give a precise scientific definition and single out a criterion for quantitative expression of machinability, it is determined using a set of criteria or functions of machinability. Functions of machinability are used to express main machinability properties of a certain structural material.

According to Heisel and Troger (2001), set of basic functions of machinability consists of:

Function of tool durability “T”
\[ T = T(x_i), \quad i=1, 2, 3, \]

Function of cutting force “Fc”
\[ F_c = F_c(x_i), \quad i=1,2,3, \quad c=1,2,\ldots, \]

Function of surface quality after machining “R”
\[ R = R(x_i), \quad i=1, 2, 3, \]

Function of accuracy of quality characteristics “X”
\[ X = X(x_i), \quad i=1, 2, 3, \]

This paper will discuss the effect of the elements of the machining regime on the cutting force, which is one of the main functions of machinability.

MATERIAL AND METHODS

This research deals with ways of determining numerical values of parameters of machinability \( C, x, y \) during the process of machining oak wood by longitudinal routing, provided that certain machining conditions are fulfilled. These conditions are expressed by the equation:

\[
F_0 = C_F \cdot a^x \cdot f_z^y
\]  

where: \( F_0 \) is tangential cutting force (N)  
\( C_F \) is machinability parameter (N.mm\(^{-2}\))
\( a \) is cutting depth (mm)
\( f_z \) is feed speed per cutting edge (mm/cutting edge)

Regression (1) is the main law that expresses mathematically the effect of so called primary group of elements or machining factors (cutting depth and feed speed) on cutting force during the process of machining oak wood by longitudinal routing.

In order to determine the factors of machinability in the function of tangential cutting force the author used multifactor orthogonal linear plan of experiment. Linear plan is used for statistical identification of the linear model by which the objective of research is mathematically expressed.
**Conditions under which the experiment is conducted**

*Machine*

Type: TABLE-MOUNTED ROUTER G-25  
Manufacturer: “Bratstvo” – Zagreb  
Shank: dia 25 mm  
Power of the engine for main movement: 3.5 kW  
Feed: feed device with cylinders; power 0.65 kW  
Tool: Cutting tool and its characteristics are shown in Fig. 1  
Diameter of router bit (D): 100 mm  
Knife thickness (s): 6 mm  
Number of cutting edges (z): 2  
Router bit width (B): 40 mm

![Fig. 1: Cutting tool, D-router bit diameter, s-knife thickness, Bb – basic plane, Sb – working plane, Ab – suspended movement plane, Kb – cutting edge measuring plane, tool cutting angles: α=12°, β=47°, γ=31°.](image1)

![Fig. 2: Cutting direction.](image2)

*Material of the work piece*

Experiment was conducted on oak wood from high stands with moisture content of 12%. Machining was conducted by conventional cutting in direction “B”, Fig 2. Machining was conducted along the narrower sides of the work piece. Fig. 3 shows a sketch of work piece whose size is 2500 x 200 x 20 mm and which has been prepared for the experiment (Fig. 4).
Components of the cutting force

Cutting power $P_c$, measured in this research was used to calculate average tangential force $F_0$. According to Ettelt (1987) and Maier (2001) tangential force can be calculated using the following formula:

\[
P_c = M \times \omega
\]

\[
P_c = F_0 \times r \times \frac{2 \times \pi \times n}{60}
\]

\[
P_c = \frac{F_0 \times r \times \pi \times n}{30}
\]

To convert tool radius from (mm) in (m) we will divide formula by 1000, so the formula can be written as:

\[
P_c = \frac{F_0 \times r \times \pi \times n}{30 \times 1000}
\]

\[
F_0 = \frac{P_c \times 0.955 \times 10^4}{r \times n}
\]

where: $P_c$ - average useful cutting power (W)
$w$ - angle speed (rad.s\(^{-1}\))
$M$ - momentum (Nm)
$F_0$ - tangential force (N)
$r$ - tool radius (mm)
$n$ - number of revolutions per minute (min\(^{-1}\))
Measuring equipment

According to Stanić et al. (1983), numeric values of the cutting power (P) for the applied machining regime were measured using the system of electrical measuring instruments, Fig. 5.

![Scheme of the system of electrical measuring instruments.](image)

Fig. 5: Scheme of the system of electrical measuring instruments.

Plan of the experiment

According to mathematical theory of experiment, two-factor functions of tangential cutting force can be expressed in the following way:

\[ F_o = C_f \cdot f_1^{p_1} \cdot f_2^{p_2} \]  

(3)

where:
- \( F_o \) - output data of experimental research (results of measurement),
- \( f_1 \) and \( f_2 \) - input data of experimental research
- \( C, p_1 \) and \( p_2 \) - parameters of machinability.

From the machinability point of view, function (3) represents a mathematical model of the mechanics (kinetostatics) of the process of machining by routing. It is used to express one of the set of characteristics of machinability.

Number of experiments needed in order to determine the value of machinability parameters in the equation (3) is determined using the following formula:

\[ N = 2^k + n_0 \]  

(4)

The above formula applies provided that four combinations (\( n_0 \)) of values of the factors \( f_1 \) and \( f_2 \) are identical, i.e. the experiment is repeated four times, each time with the same values for \( f_1 \) and \( f_2 \). Other four combinations of values of the two factors lie on the limits of the variation interval of these factors.

Before measuring the values of cutting power, it is necessary to set the limits of variation interval of factors \( f_1 \) and \( f_2 \). However, when setting the limits of variation interval of factors \( f_1 \) and \( f_2 \), the following condition must be satisfied:

\[ f_1^{2\text{sr}} = f_{\text{min}} \cdot f_{\text{max}} \quad i = 1.2 \]  

(5)

Linear, two-factor orthogonal plan with total number of experiments \( N=2^2 + 4=8 \) was adopted as the plan of experiment.

Before the beginning of the experiment, factors in the model (1) [feed per edge (\( u \)) and routing depth (\( a \)], at constant cutting width \( b=20 \text{ mm} \) and cutting speed \( v=35.6 \text{ m.s}^{-1} \) were chosen.
so that the following conditions are satisfied

\[ f_{sr}^2 = f_{max} \cdot f_{min}, \quad a_{sr}^2 = a_{max} \cdot a_{min}. \] (6)

Tab. 1 shows numerical values of factors (u) and (a).

<table>
<thead>
<tr>
<th>Factors</th>
<th>( f (\text{m.min}^{-1}) )</th>
<th>( fz (\text{mm.z}^{-1}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>max</td>
<td>4.5</td>
<td>12.5</td>
</tr>
<tr>
<td>min</td>
<td>2.0</td>
<td>0.92</td>
</tr>
<tr>
<td>medium</td>
<td>3.0</td>
<td>7.5</td>
</tr>
</tbody>
</table>

The experiment was conducted under the regimes which are defined in the plan-matrix, Tab. 2.

<table>
<thead>
<tr>
<th>Number of the experiment</th>
<th>Factors</th>
<th>Results of the experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( a (\text{mm}) )</td>
<td>( f (\text{m.min}^{-1}) )</td>
</tr>
<tr>
<td>1</td>
<td>4.5</td>
<td>12.5</td>
</tr>
<tr>
<td>2</td>
<td>3.0</td>
<td>7.5</td>
</tr>
<tr>
<td>3</td>
<td>3.0</td>
<td>7.5</td>
</tr>
<tr>
<td>4</td>
<td>4.5</td>
<td>4.5</td>
</tr>
<tr>
<td>5</td>
<td>3.0</td>
<td>7.5</td>
</tr>
<tr>
<td>6</td>
<td>2.0</td>
<td>12.5</td>
</tr>
<tr>
<td>7</td>
<td>2.0</td>
<td>4.5</td>
</tr>
<tr>
<td>8</td>
<td>3.0</td>
<td>7.5</td>
</tr>
</tbody>
</table>

Mathematical analysis of the results of the experiment

Mathematical analysis of the results of the experiment (regression analysis) includes calculation of numerical values \( C, p_1 \) and \( p_2 \) in the mathematical model (3).

Regression and dispersion analysis of the results of the experiment starts from the plan-matrix which is developed according to the properties of multi-factor plan \( 2^k + n_0 \). However, it is necessary to transform the function defined by the equation (3) into the linear form by taking its logarithm

\[ \ln F_o = \ln C + x_1 \ln a + y_1 \ln u \quad \text{or} \]

\[ y = b_0 + b_1 x_1 + b_2 x_2 \] (7)

where:

\[ y = \ln F_o, \quad b_0 = \ln C, \quad b_1 = \ln a, \quad b_2 = \ln u \]

\[ x_1 = x_f, \quad x_2 = y_f \] (8)
Calculation of coefficients $b_0$, $b_1$ and $b_2$ in the equation (8) is simplified by the introduction of new independent variables $x_1$ and $x_2$ at three levels (+1, 0, -1) using the equation of transformation:

$$x_i = 1 + \frac{2}{\ln f_{\max} - \ln f_{\min}} \ln f_{\max} - \ln f_{\min}$$  \hspace{1cm} (9)

Based on the equation of transformation, fulfilling the condition (3), the following relations between the natural and coded values of the factors are established:

$$f_i = f_{\max} \quad x_i = +1; \quad f_i = f_{\text{int}} \quad x_i = 0; \quad f_i = f_{\min} \quad x_i = -1$$

Plan matrix $(2^k + n_o = 2^2 + 4)$ formed for coded values is shown in Tab. 3. The sequence of the experiment is based on the random-number theory, and the matrix is combined in such a way that it satisfies the condition of orthogonality.

$$\sum_{i=1}^{8} x_{ix} \cdot x_{ij} = 0 \hspace{1cm} i, j = 0, 1, 2, 3 \hspace{1cm} i \neq j$$  \hspace{1cm} (10)

Tab. 3: Plan matrix for coded values.

<table>
<thead>
<tr>
<th>Number of the experiment</th>
<th>Code factors</th>
<th>Results of the experiment $y=\ln F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>+1</td>
<td>+1</td>
</tr>
<tr>
<td>2</td>
<td>+1</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>+1</td>
<td>+1</td>
</tr>
<tr>
<td>4</td>
<td>+1</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>+1</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>+1</td>
<td>-1</td>
</tr>
<tr>
<td>7</td>
<td>+1</td>
<td>-1</td>
</tr>
<tr>
<td>8</td>
<td>+1</td>
<td>0</td>
</tr>
</tbody>
</table>

Based on the property of orthogonality, we can establish the equations for calculation of the values of coefficients from the system of coefficients $b_i$ ($i=0,1,2$) in the model (10). That way, we obtain the system of general equations:

$$b_i = \frac{1}{N} \sum_{x=1}^{n} x_{ix} y_{ix} \hspace{1cm} i = 0, 1, 2$$  \hspace{1cm} (11)

which can be expressed in the following way, with respect to plan matrix from Tab. 3:

$$b_0 = \frac{1}{8}(y_1 + y_2 + y_3 + \ldots + y_8)$$

$$b_1 = \frac{1}{4}(y_1 + y_2 + y_3 + y_4) \quad b_2 = \frac{1}{4}(y_5 + y_6 + y_7 + y_8)$$  \hspace{1cm} (12)

After having identified the model (12) that corresponds to the established machining conditions, we can use the equation of transformation to transform it into its original form (3). Constant $C$ and exponents $p_i$ ($i=1, 2$) of this model are calculated using the following formula:

In order to determine the reliability of the model two criteria are used: estimate of statistical significance and test of adequacy of the model.
Estimate of statistical significance of the model parameters

Estimate of statistical significance of the model parameters \( b_i \) (\( i = 0, 1, 2 \)) is conducted according to the F-criterion, i.e. the following condition must be satisfied:

\[
F_{ri} = \frac{s_{i}^2}{s_e^2} \leq F_t \quad (14)
\]

where:
- \( F_{ri} \) - calculated value of criterion \( F \) for the parameter with index \( i \)
- \( s_{i}^2 \) - dispersion of the model parameter with index \( i \)
- \( s_e^2 \) - dispersion of results at zero-point, and
- \( F_t \) - tabular value of \( F \) - criterion under the given conditions is \( F_t = 10.13 \)

Testing of adequacy of the model

Testing model adequacy is conducted using the F-criterion, which means that if the model is adequate, the following inequality must be satisfied:

\[
F_r < F_t
\]

Calculated value of F-criterion for estimate of adequacy is obtained using the following formula:

\[
F_r = \frac{S_m^2}{S_p^2} \quad (15)
\]

where:
- \( S_m^2 \) represents the difference between the experimental values \( (y_i) \) and calculated values \( (y_i) \), calculated using the following formula:

\[
S_m^2 = \frac{1}{f_{LF}} \left[ \sum_{w=1}^{n} (y_u - y_u)^2 - \sum_{o=1}^{n_o} (y_{uo} - y_o)^2 \right] \quad (16)
\]

For level of significance \( t=0.05 \) and degree of freedom \( f_t = N-k-(n_o-1)=2 \), tabular value of the F-criterion is 9.55.

RESULTS AND DISCUSSION

Data shown in Tab. 4 below was obtained by experimental measuring of useful cutting power \( (P_c) \), by calculating the average tangential cutting force and by transforming the parameters of machinability, using the formulas described in chapter Research method.
Tab. 4: Plan matrix of factors of machinability and values obtained through the experimental measuring of average useful cutting power ($P_c$) during the machining of oak wood by longitudinal routing (direction B).

<table>
<thead>
<tr>
<th>Number of the experiment</th>
<th>Plan matrix</th>
<th>Results of experiment and calculated values</th>
<th>Model values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$P_k$ (W)</td>
<td>$F_o$ (N)</td>
</tr>
<tr>
<td>1</td>
<td>+1</td>
<td>+1</td>
<td>+1</td>
</tr>
<tr>
<td>2</td>
<td>+1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>+1</td>
<td>+1</td>
<td>-1</td>
</tr>
<tr>
<td>4</td>
<td>+1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
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<td>0</td>
</tr>
<tr>
<td>6</td>
<td>+1</td>
<td>-1</td>
<td>+1</td>
</tr>
<tr>
<td>7</td>
<td>+1</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>8</td>
<td>+1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Values of parameters $b_i$ were calculated using the formulas (12) given in the chapter “Material and methods” by using the data obtained through the experimental measurement.

$$b_0 = \frac{1}{8}(2.8736+2.3702+2.0669+2.3125+2.2721+2.4069+1.8871+2.3608)$$

$$b_0 = 2.3188$$

$$b_1 = \frac{1}{4}(2.8763+2.0669-2.4069-1.8871)$$

$$b_1 = 0.1623$$

$$b_2 = \frac{1}{4}(2.8736-2.0669+2.4069+1.8871)$$

$$b_2 = 0.3316$$

After inputting the values of parameters $b_i$ ($i=1,2$) in the model (8) the following equation is obtained:

$$y = \ln F_o = 2.3188 + 0.1623x_1 + 0.3316x_2$$

If we switch from coded coordinates to natural coordinates using the equation of transformation (13), model (3) is obtained:

$$p_o = b_0 + b_1 + b_2 - (p_1 \ln a_{max} + p_2 \ln u_{z max})$$

$$p_o = 2.2657$$

$$p_1 = \frac{2 \cdot b_1}{\ln a_{max} - a_{min}}$$

$$p_1 = 0.4$$

$$p_2 = \frac{2 \cdot b_2}{\ln u_{z max} - u_{z min}}$$
Inputting the values of the constant $C_{F_0}$ and the exponents x and y in the model (1) we obtain regression relation of the average tangential cutting force at the longitudinal machining of oak wood by routing to cutting depth $(a)$ and to feed per edge $(u_x)$ for the given experiment conditions which can be expressed through the following formula:

$$F_0 = 9.64 + 0.65 \cdot f_z^{0.45}$$

(17)

When values of $a$ and $u_x$, according to plan matrix are input in the above formula, Tab. 2, we obtain model values of average cutting force, whose values are input in plan-matrix, Tab. 4.

Estimate of statistical significance of the model parameters was obtained according to F-criterion, according to which, the condition that calculated value of F-criterion must be greater than the tabular value must be fulfilled $(F_{cal} > F_{t}; F_{t1} > F_{t}; F_{t2} > F_{t})$. In our case that condition is satisfied $(F_{cal} = 18864 \geq 10.13; F_{t1} = 46.2 > F_{t} = 10.13; F_{t2} = 192.9 > F_{t} = 10.13)$.

The condition of statistical significance is fulfilled for all parameters.

Testing of adequacy of the model is conducted using the F-criterion, i.e. if the model is adequate, condition $F_{cal} < F_{t}$ $(F_{cal} = 3.92; F_{t} = 9.55)$ must be fulfilled. This means that the condition of adequacy is fulfilled so mathematical model $F_0 = 9.64 + 0.65 \cdot f_z^{0.45}$, with high enough accuracy, describes the relation of average tangential force to the values of given factors under the specified conditions of experiment.

![Diagram](image)

**Fig. 6:** Diagram of relation of average tangential cutting force $(F_0)$ to the cutting depth $(a)$.

**Fig. 7:** Diagram of relation of average tangential cutting force $(F_0)$ to the feed per edge $(f_z)$.

Diagram of relation of average tangential cutting force during the process of machining oak wood by longitudinal routing to the cutting depth $(a)$ and feed per edge $(f_z)$ for the given conditions of experiment is shown in the double logarithmic system in Figs. 6 and 7.
CONCLUSIONS

Analyzing the position of straight lines in the given diagrams and the values of the exponents in the model (17) the following can be concluded:

• Average tangential cutting force increases with the increase of any of the analyzed factors;
• With regard to intensity, cutting depth has a greater effect on the cutting force;
• If the cutting depth increases, the increase of cutting force is faster when feed per edge is greater.
• If the feed per edge is increased, the increase of tangential cutting force is more significant when the value of cutting depth is greater.

REFERENCES


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