DIVERSE INFLUENCE OF USER-ECONOMIC ASPECTS TO TRUSS AND RAFTER ROOF SYSTEMS AND THEIR COMPARISON

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ABSTRACT

Article deals with a problem of limited attic usability of metal plate fastened truss roofs. It defines an approach how to learn a capacity of attic due to the varying shape of roof and depending on a roof pitch. The article also takes a look at the pros and cons of trussed and rafter roof construction and compares them through a customer perspective. Arched roof offers the biggest volume of all kinds of shapes, although its usability is not better than gabled shape when evaluating other aspects such as design difficulty or material consumption. When the priority is an attic utilization only, there is no difference of choosing classic bound rafter roof or trusses if span is not bigger than 10 meters, but it is important to evaluate all user-economic criteria. When span exceeds 10 meters, the trussed roof is recommended option. To get the maximal usability of gable roofs, we should also follow its proper height from 3.3 to 3.4 meters.

KEY WORDS: Truss, room in attic, roof, span, ideal pitch of roof.

INTRODUCTION

The fact of the matter is that the roof is one of the principal causes of energy loss in constructions. In consequence we can observe obvious effort to decrease the roof surface to minimum. The indisputable advantage of this approach is also material saving (Hudec et al. 2013).

The approach of roofing was changed after roof trusses with metal plate fasteners entered market (Jelinek 2008, Karadelis 2000). Hudec et al. (2013) say that their manufacturing became popular very quickly because it is easier and faster. Since the beginning, trusses served primarily for big span buildings such as warehouses and big depots (El-Sheikh and Shaaban 1999). However, people began to use it more and more often for roofing of residential houses. The
massive expansion of using the trusses came hand in hand with the change of lifestyle and the reduced need to store as many stuff as before. That is the one of the reasons why the single storey bungalow type of houses had become so popular. The most efficient way of a roofing in this case is to choose a flat roofing, nonetheless sometimes it does not fit into a communal architecture of the buildings around (Hudec et al. 2013, Carter 1997). It can force us to choose the other shapes which offer an extra space. The potential of additional should not be overlooked. It is very important to consider this issue when designing a roof construction, because it is not so easy to cut off some trusses in the attic when they are already fabricated.

That leads us to the fact that a customer should consider more aspects, not just the price (Kuklík 2005). Basically, customers have to deal with three basic criteria and make a priority scale. Firstly if we try to make the most of loft space, then we talk about the user aspects. Secondly in the case that the customer wants to save as money as possible, then we must think about the aspect of price – economic aspect. At last as much money as not least there is an aspect of time which is undoubtedly important as well and closely related to previous one. There are also other aspects which should not be neglected for example transport to the construction site and possible storage (Blass 1995a). The optimal roof construction would be naturally that one which meets all these requirements in the best possible way. The challenge for the suppliers is to be well prepared for any kind of customer’s demand. To make it properly, we have to split all aspects and solve them one by one to make a final consideration which contains the best combination of all these aspects. Only in this way we could achieve the high quality supply and make customers satisfied.

**MATERIAL AND METHODS**

The Gang nail truss is considered as the most widely used type of truss. Its joints are provided by steel plate fasteners (Karadelis 2000, Kuklík 2005). This type of joint facilitates easy prefabrication. Its stiffness is very high because it is a type of surface fastener which does not lower wooden elements (Silih et al. 2005). The usage of other types of trusses is minimal and all calculations in this article co deal with the steel plate connectors.

The most popular shapes of the roofs built today are both gable roof and hip roof (Hájek and Filipová 1997). They have some pros and cons. In the case of gable roof shape it is easier to design and calculate what is advantageous, but there is a need to build gable construction on the each side of a building which is not needed if we build a hip shape of roof. The hip shape has also a good look and the construction is more accessible. On the other hand it is not smart to build the hip shape in mountain area, because there is a recommendation about having a big pitch which allows the snow to slide down. Another big disadvantage of the hip roof is a cramped room in attic. Despite all the disadvantages these types of roof are still considered as the most popular in Central Europe. Their only difference is that the hip roof has four pitched sides of the roof, so it does not have the gables as the gable roof (Wacker 2010) nevertheless if we cut a building in the middle of its length they both have the same cross-section area. It is mostly the span which is object of interest in this article. That is why the calculations are made for the gable roof shape only.

To provide reasonable comparisons we need to have uniform conditions to obtain valuable results. To find out which of the shapes is the most efficient we set up our own method. We introduced new terms such as theoretical attic volume, useable attic volume and finally usability which is based on ratio of previous two. Theoretical attic volume is defined as a space bordered by the roof shell, excluding roof construction itself. According to rules in standard ČSN 73 4301
2004, we even could not include all theoretical volume as usable we had to deduct the substandard parts so we obtained a real usable volume. To create an objective comparison we set up the same fundamental conditions. All roof shapes were roofed with the same platform and with the same roof overhang, even height of the roof had to be the same in all cases except arched roofs, because we had to meet preliminary design (Steel Truss 2003). This tentative preliminary design says that ratio between roof height and span should be at least 1/6 for triangle roofs and 1/8 – 1/6 for arched ones (Blass 1995b, Koželouh 2004, Silih 2005).

We also have to meet all the most important standardized rules such as minimum height of the knee walls in attic, equals 1 300 mm, minimum height of the ceiling has to be 2 300 mm. Clearance height has to be at least above one half of a residential room (ČSN 73 4301 2004). Because our result has to be usable in practice we have to express our equation relating to dimension measurable at design stage (Underwood et al. 2001). It is a problem in case of measurement “a” which represents the internal length of the bottom chord which we can measure after design only. That is why we also have to create a way to calculate it from “L” which is total length of the bottom chord known in advance.

To compare the trussed and bound rafter we use the data and the equations which we gained in previous parts. We use measurements of the most efficient roofs that we calculated. Because there is supposed to be a linear relationship between length and price, the span is a factor which causes exponential price growth. That is a reason why the exclusion of the length from those equations is essential. Price of wood changes very often, hence in graphs and tables we compare material consumption for both trussed as well as basic rafter systems. Our comparison is situated at family houses with a span from 6.7 up to 11.5 meters.

To compute the most effective roof shape we have to set up an uniform conditions first. Our reference building has a rectangular shape 6.5 m times 14 m. Main roof overhang is 0.9 m and height is 3.9 m. We evaluated 4 different roof shapes – gable, hip, gambrel and arched. First, we created four different equations according to the shape and introduced the theoretical attic volume \( V_{\text{teor}} \). The equations are following:

\[
\text{a) Hip roof shape}
\]

\[
V_{\text{teor}} = V_c \times \left( \frac{2a^2 + 3bl_1}{6} \right) \quad (\text{m}^3)
\]

where:
- \( V_c \) – the biggest perpendicular distance from floor to internal roof peak (m),
- \( a \) – internal length of bottom chord of the truss or joining beam (m),
- \( l_1 \) – internal length of gable part of building (m).

\[\text{Fig. 1: Dimensions of roof shapes.}\]
b) Gable roof shape

\[ V_{\text{teor}} = \frac{1}{2} \cdot V_c \cdot a \cdot l \ (m^3) \]  

where: \( V_c \) – the biggest perpendicular distance from floor to internal roof peak (m),  
\( a \) – internal length of bottom chord of the truss or joining beam (m),  
\( l \) – internal length of building (m).

c) Arched roof shape

\[ V_{\text{teor}} = \frac{r^2}{2} \cdot (\alpha - \sin \alpha) \cdot l \ (m^3) \]  

where: \( r \) – radius of arch (m),  
\( \alpha \) – central angle of roof arch (rad).

As second, we introduced real useable volume \( V_v \) by deducting substandard part according to ČSN 73 4301 2004. Minimal height of an attic in a residential buildings is 2 300 mm. Minimal height under a bevelled wall should be 1300 mm. It respects these rules in calculations and use minimal values to get maximal usage. And then we obtain the following equations:

a) Hip roof shape

\[ V_v = \left[ B \cdot h_1 \cdot l_1 \right] + \left\{ \frac{h_2}{3} \cdot \left[ \frac{(B \cdot l_1) + \left( \frac{1}{2} \left( (B \cdot L_1) \cdot (b \cdot l_2) \right) \right)}{\sqrt{h_1 + h_2}} \right] \right\} \ (m^3) \]  

where: \( B \) – width of attic space with height \( \geq 1 \ 300 \) mm (m),  
\( h_1 \) – height of knee walls (m),  
\( h_2 \) – vertical height of bevelled ceiling in the attic (m),  
\( b \) – width of attic ceiling (collar beam) (m),  
\( l_1 \) – total length of usable attic with height \( \geq 1 \ 300 \) mm (m),  
\( l_2 \) – total length of horizontal attic ceiling (m)

Standard given conditions for residential buildings:

\( h_1 \geq 1 \ 300 \) mm ; \( H = (h_1 + h_2) \geq 2 \ 300 \) mm

b) Gabled roof shape

\[ V_v = \left( B \cdot h_1 + \frac{(B+b) \cdot h_2}{2} \right) \cdot l \ (m^3) \]  

c) Arched roof shape

\[ V_v = \left[ B \cdot h_1 + \left( \frac{B+b}{2} \right) \cdot h_2 \right] \cdot l \ (m^3) \]  

d) Gambrel roof shape

\[ V_v = \left[ B \cdot h_1 + \left( \frac{B+b}{2} \right) \cdot h_2 \right] \cdot l \ (m^3) \]  

where: \( B \) – width of attic space with height \( \geq 1 \ 300 \) mm (m),  
\( b \) – width of attic ceiling (collar beam) (m),  
\( h_1 \) – height kneewalls in the attic (m),  
\( h_2 \) – vertical height of bevelled ceiling in the attic (m),  
\( l \) – internal length of building (m).
Standard given conditions for residential buildings:

\[ b_1 \geq 1300 \text{ mm}; \quad H = (b_1 + b_2) \geq 2300 \text{ mm} \]

The last part is to compare four different roof shapes and to compute total usability efficiency \( U_0 \) which comes from following equation. It says how much from volume under a shell of the roof is eligible. It is very important to know aforementioned because of the heat lose or the waste of material during building phase.

\[
U_0 = \frac{V_v}{V_{teor}} \times 100 \quad (\%)
\]  \hspace{1cm} (8)

If we take a closer look to usability of a room in the attic we can see that, according to varying span of a building ideal, pitch varies too. That is why we created tables and drawings for spans from 6.7 up to 11.5 meters and for a pitch from 30 to 45 degrees. After that we implemented the rules from standard ČSN 73 4301 2004. By that we got ideal pitch for each span of buildings with 0.3 m intervals, it means seventeen different spans and fifteen angles. For spans lower than 9 meters we excluded small angles because they did not offer room in attic which is wide enough for residential use. We used a graphical comparative method in combination with equations we set up before. The drawings of houses for every span and angle were made in the AutoCAD software and were provided with required dimensions, as we can see for illustration in Fig. 2.

\[ S_{teor} = \frac{1}{2} \times V_c \times a \]  \hspace{1cm} (9)

where: \( V_c \) – the biggest perpendicular distance from floor to internal roof peak (m)  
\( a \) – internal length of bottom chord of the truss or joining beam (m).

\[
S_v = B \cdot h_1 + \frac{(B + b) + h_2}{2} \quad (\text{m}^2)
\]  \hspace{1cm} (10)

where: \( B \) – width of attic space with height \( \geq 1300 \text{ mm} \) (m),  
\( b \) – width of attic ceiling (collar beam) (m),  
\( h_1 \) – height kneewalls in the attic (m),  
\( h_2 \) – vertical height of bevelled ceiling in the attic (m).

Fig. 2: Graphical definition of the most efficient pitch.
Standard given conditions for residential buildings:

\[ h_1 \geq 1300 \text{ mm} \; ; \; H = (h_1 + h_2) \geq 2300 \text{ mm} \]

Because the value “a” is not measurable at the beginning of the designing phase we have to be able to compute it from the total span of truss or rafter L. Because L is varying depending on overhang and dimensions of the used chords we have to use these variables to compute a universal way. Specifically we use the following equations according to Fig. 3.

\[ a = L - 2x \; ; \; x = \frac{H}{\sin a} \]  \hspace{1cm} (11, 12)

where: \( H \) – height of bottom chord

We used which we created to get value “a” from known value “L”. It was set up for dimensions “H” from 14 to 24 mm. According to “a” value we can choose the best pitch for a roof to create room in attic the most preferably.

Tab. 1: Length conversion.

<table>
<thead>
<tr>
<th>L (m) =&gt;</th>
<th>6.7</th>
<th>7</th>
<th>7.3</th>
<th>7.6</th>
<th>7.9</th>
<th>8.2</th>
<th>8.5</th>
<th>8.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>a-H14 (m)</td>
<td>No change</td>
<td>&quot;a&quot;= L - 0.3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a-H16 (m)</td>
<td>9.1</td>
<td>9.4</td>
<td>9.7</td>
<td>10</td>
<td>10.3</td>
<td>10.6</td>
<td>10.9</td>
<td>11.2</td>
</tr>
<tr>
<td>a-H18 (m)</td>
<td>&quot;a&quot;= L - 0.6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a-H20 (m)</td>
<td>10.8</td>
<td>11.1</td>
<td>11.4</td>
<td>11.7</td>
<td>12</td>
<td>12.3</td>
<td>12.6</td>
<td>12.9</td>
</tr>
<tr>
<td>a-H22 (m)</td>
<td>&quot;a&quot;= L - 0.9</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a-H24 (m)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

As next we choose the best case of roofing i.e. for each “a” value to calculate material consumption for trusses and compare it to material consumption of classic rafter roof in the same cases. For rafter roofs we used practical data from the roof manufacturing companies to calculate dimensions of rafters according to length and angle.

In the end we created a comparison table for trusses and rafters in four span categories. It allows us to evaluate all aspects at once. Every aspect has a different importance so we created the scale pursuant to economic influence of each aspect to final potential customer. The price of the final building and the comfort of its users is influenced by all the aspects. That is why we can divide them to user aspects and economic ones. Economic aspects can influence expenses directly or indirectly. Especially, indirect expanses are not considered properly when the roof is being chosen because they are not visible at first sight. Retaining wall and ceiling function or installation speed are then considered as indirect economical aspects because they cannot be seen on any invoice but have a reasonable impact on a total price of the building. On the other hand, material consumption, prefabrication and aspects of the employees’ skills directly influence the price of the roof system. In contrast, user aspects are related to the price only slightly, but their major relation is with living environment.
RESULTS AND DISCUSSION

As we can see in Tab. 2 the best usability (approximately 80 \%) has reached the arched and the gambrel shape. The well designed gable shape has the usability about 75 \% and maximum usability of hip roof goes up to 70 \%. This outcome seems to be not surprising result however in the case of arched and gambrel shapes, we can optimize more variables than in the case of gable and hip roof where the unique option is pitch. The most effective criteria are the shape and the pitch of the roof (Teitel and Wenger 2010). That is why optimal pitch is crucial in regard to maximize efficiency of the gable roofs. This particular point is also supported by the results of Heimann (2006).

Tab. 2: Roof shapes usability.

<table>
<thead>
<tr>
<th>Shape</th>
<th>Usability (%)</th>
<th>Shape</th>
<th>Usability (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arched</td>
<td>75-80</td>
<td>Gable</td>
<td>70-75</td>
</tr>
<tr>
<td>Gambrel</td>
<td>72-77</td>
<td>Hip</td>
<td>65-70</td>
</tr>
</tbody>
</table>

\[ P = -2.99 \cdot a + 63.99 \]  
(13)

where:  
P – pitch,  
a – corrected span (according the correction table above).

In the case of room in attic trusses, we have to expect some bending moments because the top chords are not fully supported by internal bars. In Tab. 3 we can see maximal unsupported length of top chord. This value does not exceed two meters. It is good because based on that we can use smaller cross-section of top chords. The ratio between height and span also meets the preliminary design of Blass (1995a) which says that ratio between height and span of the triangular roof.
should be min. 1/6. Relation of length of building and consumption of material is obviously linear however with growing span material consumption grows slightly exponentially. Nonetheless Fig. 4 shows that if we have the right pitch for an individual span, we obtain a linear related values as Tab. 3 shows for maximal use of attic space the height of gabled roof should be in interval from 3.3 to 3.4 meters. Fig. 4 proves that consumption of material of rafter roofs is lower until span which does not exceed 9 meters. Then both truss and rafter roofs are almost equal but in the case of span longer than 10 meters trussed roofs prove to be noticeably more favourable.

Tab. 3: Ideal roof pitch characteristics.

<table>
<thead>
<tr>
<th>Span (a) (m)</th>
<th>6.7</th>
<th>7</th>
<th>7.3</th>
<th>7.6</th>
<th>7.9</th>
<th>8.2</th>
<th>8.5</th>
<th>8.8</th>
<th>9.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pitch (°)</td>
<td>45</td>
<td>44</td>
<td>42</td>
<td>41</td>
<td>40</td>
<td>39</td>
<td>38</td>
<td>37</td>
<td>36</td>
</tr>
<tr>
<td>Roof height (m)</td>
<td>3.35</td>
<td>3.38</td>
<td>3.29</td>
<td>3.30</td>
<td>3.31</td>
<td>3.32</td>
<td>3.32</td>
<td>3.32</td>
<td>3.31</td>
</tr>
<tr>
<td>Usability (%)</td>
<td>75.12</td>
<td>75.00</td>
<td>75.03</td>
<td>75.29</td>
<td>75.25</td>
<td>75.23</td>
<td>75.23</td>
<td>75.24</td>
<td>75.27</td>
</tr>
<tr>
<td>Maximal unsupported length of top chord (m)</td>
<td>1.41</td>
<td>1.44</td>
<td>1.47</td>
<td>1.52</td>
<td>1.56</td>
<td>1.59</td>
<td>1.62</td>
<td>1.66</td>
<td>1.70</td>
</tr>
</tbody>
</table>

Tab. 4: Scaled comparison of truss and rafter roofs.

<table>
<thead>
<tr>
<th>Importance Scale</th>
<th>Span (m) →</th>
<th>6-9</th>
<th>9-10</th>
<th>10-12</th>
<th>12 and more</th>
</tr>
</thead>
<tbody>
<tr>
<td>Considered Aspect ↓</td>
<td>T</td>
<td>R</td>
<td>T</td>
<td>R</td>
<td>T</td>
</tr>
<tr>
<td>2 Architecture variability</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>5 Attic usability</td>
<td>4</td>
<td>5</td>
<td>4</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>3 Roof shapes variability</td>
<td>5</td>
<td>4</td>
<td>5</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>3 Installation speed</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>4 Ceiling function</td>
<td>5</td>
<td>1</td>
<td>5</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>3 Retaining wall</td>
<td>5</td>
<td>3</td>
<td>5</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>5 Material consumption</td>
<td>4</td>
<td>5</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>4 Prefabrication</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>2 Employee skills needs</td>
<td>5</td>
<td>4</td>
<td>5</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>145</strong></td>
<td><strong>128</strong></td>
<td><strong>145</strong></td>
<td><strong>103</strong></td>
<td><strong>134</strong></td>
</tr>
</tbody>
</table>

Final comparison of the truss (T) and rafter roofs (R) could be seen in Tab. 4. After the proper examination of all the important aspects we can see that trusses are more convenient choice in every span category. The importance of every category is based on the share of the final price in case of direct economical aspects (last 3), regarding indirect economical aspects it
is based on possible savings (middle 3) and in the case of user aspects (first 3) it is estimated of
needs what is the main clue. Trusses succeeded in the most important aspects. Especially in the
case of economical aspects such as attic usability, ceiling function, material consumption and
prefabrication they seem as excellent system of roofing, just says (Sui et al. 2013). Because of these
facts trusses are considered as better choice than rafter constructions; however we cannot say that
this statement is right in every situation. The biggest difference is observed in the cases where the
span is longer than 9 meters. Under this limit, classic rafter system can be more efficient in some
cases because importance scale can be slightly changed by every single customer.

CONCLUSIONS

The effective usage of attic volume can be sometimes very difficult. The biggest efficiency
has the arched shape of roof it can be more than 80 % if the design is made well. The arched
shape type of roof is followed by gambrel type with 77 % of efficiency, gable type with 75 % and
hip type with 70 % of efficiency. If the rules from standards are combined into the design, the
ideal pitch of gable and hip roofs is variable with changing span in linear relation according to
which we can easily define the most efficient pitch of roof in terms of attic usability. If the proper
pitch is set the usability is not changing with the growing span of the building until reaches about
12 meters. Also interesting fact is that if the height of roof is maintained between 3.3 and 3.4
meters, the biggest efficiency of attic use of space will be obtained. That is also in accordance to
preliminary standards’ design which also secures simple transport.

Proper pitch in accordance to span of building also shows that consumption of material is
linear in case of low-span buildings. Regarding to linear trend of consumption of material it was
proved that classic rafter roofs have steeper trend in comparison to trusses and they both meet
when value of span is about 9 meters. Simultaneously, if the span reaches limit of 10 meters,
the difference in consumption of material becomes to be very significant. After evaluation of
all important criterions the truss could be better choice even in the case of span smaller than 9
meters.

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