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ANALYSIS OF SUPPORT OF STAIRS IN A WOODEN PREFABRICATED STAIRCASE WITH ONE-SIDED SUSPENDED STAIRS MADE FROM SCOTS PINE (*PINUS SYLVESTRIS*) WITH THE USE OF EXPERIMENTAL TESTS AND NUMERICAL ANALYSES

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ABSTRACT

The article describes experimental tests of a detail of a wooden prefabricated staircase with one-sided suspended stairs made from Scots pine (*Pinus sylvestris*) in the wall with steel support bars. It is a detail under high stress which was identified by a 3D numerical analysis performed with the use of finite element methods using the calculation system ANSYS. It contains a comparison of the results from experimental tests and numerical 3D analyses. In addition, it describes material models used with the numerical 3D analysis (rectangular orthotropy and cylindrical anisotropic plasticity) as well as non-interactive (maximum stress criterion) and interactive (Hoffman and Tsai-Wu criteria) failure criteria for prediction of the occurrence and propagation of failures in wood.

KEYWORDS: Scots pine (*Pinus sylvestris*), staircase, elastic and material constants, rectangular orthotropy, cylindrical anisotropic plasticity.

INTRODUCTION

When designing staircases, it is necessary to deal with the requirements that concern the conditions of safe walking on stairs (Veselý and Mikš 2010). A designed staircase should be comfortable for users and should provide proportionality, regularity, rhythm and order (Veselý and Mikš 2010). When designing staircases, it is necessary to correctly design the dimensions of the staircase area, construction system of the staircase, dimensions of stairs, stair shapes, etc. in accordance with ČSN 73 4130 (2010), while taking into account the scheme of the building and economic factors.

Suitably selected construction systems of the staircase, or staircase types respectively, contribute to elegance, originality and unique style of the building. Therefore, subtle, in maximum degree lightweight and airy, staircases with attractive and modern look are currently designed. An example of such staircase is a staircase with one-sided suspended stairs without a string or with a wall string (Fig. 1).

a) b) c)

Fig. 1: Wooden prefabricated staircase with one-sided suspended stairs with a wall string a) and without wall string b), c). Source from (JEMA Svitavy 2006).

The project MPO ČR IMPULS, registration number FI-IM2/053, titled "Research and Development of a new generation of staircases to residential and civil buildings" numerically and experimentally analysed the structure of a wooden prefabricated staircase with one-sided suspended stairs without a wall spring made form Scots pine including its details. The analyses aimed to design changes that would improve versatility and usability of the segment staircase and increase variability of the construction system in combination with reduction of costs on the staircase production with material savings.

MATERIAL AND METHODS

The numerical analysis of a wooden staircase with one-sided suspended stairs in accordance with ČSN EN 1991-1-1 (2004) was performed on a straight staircase made from Scots pine with construction height of 3.00 m, aligned span of 4.86 m and ground distance of 3.98 m (Pěnčík and Lavický, 2011) and (Pěnčík et al. 2013), (Fig. 2). Dimensions of stairs without risers of 0.9 m comply with the requirements of a Czech design standard ČSN 73 4130 (2010) for residential houses. The width of stairs at the walking line was designed to 0.314 m, with the stairs overlap of 0.06 m. The thickness of stairs of 0.05 m was designed taking into account the existing production methods (JEMA Svitavy 2006). The dimensions of the handrail and newels of profile 0.14 × 0.05 m were designed to be made from single-layer boards from solid wood. At the outer side, the stairs were suspended with the use of a system of steel bars (24 pieces) of a profile of ⌀ 12/2 mm to the massive handrail, which was taken along the outer side of the whole staircase. Each stair was suspended on three bars (Fig. 2) and was connected with the previous and the following stair with the use of wood distance elements. At the wall side, the stairs were placed with the use of 2 steel support bars which were embedded in the wall through rubber cases.

To analyse the selected geometric arrangement of the prefabricated staircase, a calculation system ANSYS was used (ANSYS 2011), which was used for the creation of a 3D analysis model. The analysis model (Fig. 2) was created with the use of 3D finite elements of type SOLID45, SOLID92, SOLID95 and SURF154 (ANSYS 2009). The method and description of modelling the parts of the staircase are specified in (Pěnčík and Lavický 2011) and (Pěnčík et al. 2013). The aim of the numerical analysis was to identify staircase spot with higher stress for their subsequent detailed experimental analysis.

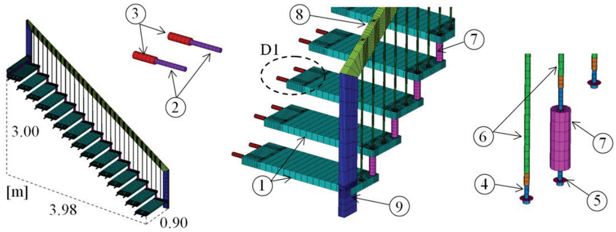

Fig. 2: 3D analysis model – 1) stairs, 2) steel support bars, 3) rubber cases, 4) connection bolts, 5) mats for bolts, 6) steel bars, 7) distance elements, 8) massive handrail, 9) newel.

When modelling the wooden parts of the staircase, i.e. stairs, newels, handrails, and distance elements providing the height level of stairs at the side of the handrail, produced from single-layer boards from solid wood, the wood was considered as homogeneous material, i.e. neglecting the effects of early and late wood, and the wood was described by the rectangular orthotropy material model (Bergman et al. 2010), (Bodig and Goodman 1973), (Bucur 2006), and (Mascia and Lahr 2006). In order to predict the wooden material failure, Hoffman (Camanho 2002), (Hoffman 1967), (Schellekens and De Borst 1990) a Tsai-Wu (Camanho 2002), (Danielsson and Gustafsson 2013), (Tsai and Wu 1971) failure criteria were used; both described by a relationship 1) written in a tensor form. The relationship 1) was modified according to (Bodig and Goodman 1973) and (Tsai and Wu 1971) neglecting the component F_{ijk} and simultaneously neglecting the components out of the main diagonal. The condition of failure expressed by relationship 1) was possible to write in its shorter version 2), where σ_1 to σ_6 are arranged components of the stress vector $\{\sigma\}$ and F_i, F_{ij} components of the polynomial mentioned in (Camanho 2002) and (Tsai and Wu 1971).

$$F_i\sigma_i + F_{ij}\sigma_i\sigma_j + F_{ijk}\sigma_i\sigma_j\sigma_k \geq 1,\ i, j, k = 1 \ldots 6\ (\text{for 3D}) \tag{1}$$

$$\begin{aligned} &F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + F_{33}\sigma_3^2 + F_{44}\sigma_4^2 + F_{55}\sigma_5^2 + F_{66}\sigma_6^2 \\ &\quad + 2F_{12}\sigma_1\sigma_2 + 2F_{13}\sigma_1\sigma_3 + 2F_{23}\sigma_2\sigma_3 + F_1\sigma_1 + F_2\sigma_2 + F_3\sigma_3 \geq 1 \end{aligned} \tag{2}$$

The numerical analysis found that increased stress concentration and loading of construction parts of the prefabricated staircase with one-sided suspended stairs occur at the stairs in their supporting in the wall with the use of steel support bars (Pěnčík and Lavický 2011, Pěnčík et al. 2013, Pěnčík 2013). The place is shown in (Fig. 2) as place D1).

MATERIAL AND METHODS

In order to verify behaviour and determine the maximum bearing capacity, the experimental tests were performed in a laboratory of Institute of Building Testing, Faculty of Civil Engineering, Brno University of Technology within the project HS 12625002 (Pěnčík et al. 2006). The detail

(D1) of supporting of the stair in the wall by steel support bars (Fig. 3) was experimentally tested. 3 specimens were tested in the tests of this detail.

An 8-channel measuring switchboard HBM Spider 8 (2006), which was connected to PC, was used for the experimental tests, in order to continuously record the measured data, i.e. amount of the loading and vertical displacement of selected points in time, under the frequency of data saving of 5 Hz, i.e. in the time interval of 0.2 s. The vertical displacement of the selected points was monitored with the use of inductive displacement transducer sensors of HBM WA-T/50 mm (2012c) with the measuring range of 0 to 50 mm and accuracy of 0.001 mm, and with the use of potentiometric trajectory sensors MS04 with the measuring range of 0 to 120.8 mm and accuracy of 0.05 mm.

The scheme of the arrangement of the test of stair supporting in the wall with the use of steel support bars is shown in (Fig. 3a). The tested specimens, marked as ST2, ST3 and ST4, were placed on the loading track (Fig. 3b). Loading of the tested specimens was applied by a hydraulic cylinder over the spread footing of the dimensions 100 × 100 mm, which was installed so that the most unfavourable effects on the tested specimens were reached. The loading was divided into loading steps by loading intensity of 1 kN. A time interval of 5 minutes was in between the loading steps.

a) b)

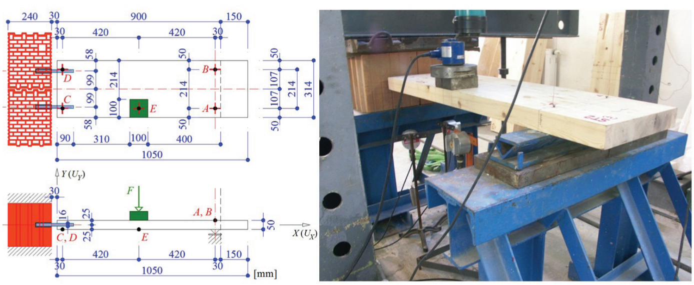

Fig. 3: Scheme of arrangement of test with marked measuring points a) and loading track with the tested specimen ST2 b).

Damage occurred with all specimens by a chip off in the place of a steel support bar that is closer to the applied load, i.e. at the steel support bar at the measuring point *C* (Fig. 3a). The magnitude of forces *F*, under which the specimens were damaged, and the corresponding vertical displacement $U_{Y,E}$ in the measuring point *E* are shown in (Tab. 1).

Tab. 1: Results of experimental tests and numerical analysis of specimens ST2, ST3 and ST4.

Experiment	**ST2**	**ST3**	**ST4**
F (kN)	6.98	5.70	8.33
$U_{Y,E}$ (mm)	5.44	6.31	9.39
$U_{Y,Em}$ (mm)	4.14	5.21	8.36

Numerical analysis	**ST2**	**ST3**	**ST4**
F (kN)	6.31	6.22	9.78
$U_{Y,E}$ (mm)	4.00	4.14	7.19

The maximum magnitude of the force necessary to damage a specimen and corresponding vertical displacement in the measuring point *E* was measured at the specimen ST4 (F = 8.33 kN; $U_{Y,E}$ = 9.39 mm). The average force magnitude, or corresponding with the average vertical displacement at the failure respectively, is F = 7.00 kN resp. $U_{Y,E}$ = 7.05 mm. The damaged tested specimens are shown in (Fig. 4). The graphic relationship between vertical displacement and the applied force, while taking into account the same scales of coordinate axes for all specimens, is shown in graphs in (Fig. 5).

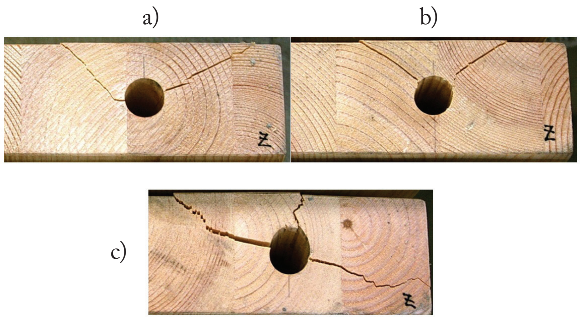

Fig. 4: Detail of failures of the tested specimens at the place of steel support bar at the measuring point C for specimen ST2 a), ST3 b) and ST4 c).

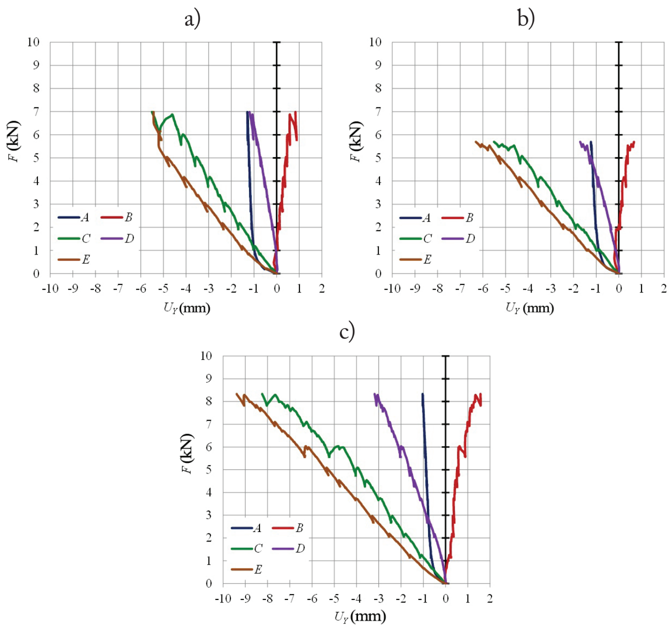

Fig. 5: Graph of relationship between vertical displacement U_Y (mm) in the measuring points and the applied force F (kN) for specimens ST2 a), ST3 b) and ST4 c)

The comparison of the arrangement of annual rings at the place of the steel support bar (Fig. 4) and the magnitude of forces at the failure (Tab. 1) clearly show that the magnitude of the force at the failure is influenced by the arrangement of annual rings. The effect of annual rings is also clear on the character of the specimen damage. The failure of specimen ST4 (Fig. 4c) is in comparison with failures of specimens ST2 and ST3 different due to the arrangement of annual rings at the place of the steel support bar.

Theoretical part

A detail of the supporting of a stair into the wall using steel support bars was, after experimental tests, also numerically analysed with respect to the partial objective of the project FI-IM2/053, which was a modification of wooden prefabricated staircases, respectively of place of excessive stress.

To analyse the behaviour of the detail of stair supporting into the wall with steel support bars, three 3D analysis models for tested specimens ST2, ST3 and ST4 (Fig. 3a) were used that had been created in calculation system ANSYS (ANSYS 2011). The analysis models were created using 3D finite elements of the type SOLID45, SOLID92, SOLID95, SURF154, TARGE170 and CONTA174 (ANSYS 2009). Using 3D finite elements of the type SOLID45, SOLID92 and SOLID95 were modelled constructional parts 1) to 5) depicted in (Fig. 6): 1) stair, 2) steel support bars, 3) rubber cases, 4) spread footing and 5) steel supporting device. Dimensions of numerical models are shown in (Fig. 3).

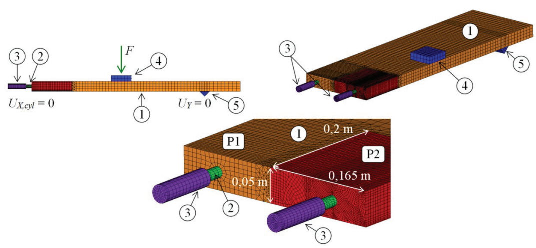

Fig. 6: 3D analysis model.

In the modelling of wooden stairs that had been made from single-layer boards of solid wood, two approaches to modelling of wooden elements were combined together – wood as homogeneous material, i.e. without distinguishing the effect of early and late wood, where wood is described by a rectangular orthotropic material model (Bergman et al. 2010), (Bodig and Goodman 1973), (Bucur 2006), (Mascia and Lahr 2006) – approach P1 and wood as a homogeneous material, where wood is described by cylindrical anisotropic plasticity model without hardening (Dinckal, 2011), (Moses and Prion 2002, 2004) – approach P2. Places where approaches P1) and P2) have been applied are marked in (Fig. 6). The approach P2) was applied for those locations of a stair where failures occurred during experimental tests, i.e. in places of the supporting of stairs on steel support bars, closer to where the loading has effect. In this area of the size of 0.165 × 0.2 m the effect of annual rings will also be considered. The orientation and arrangement of annual rings was idealized in the entire area using an arrangement of annual rings on faces of test specimens ST2, ST3 and ST4. Views on the faces of test specimens with idealized pattern of annual rings and numerical models are shown in (Fig. 7).

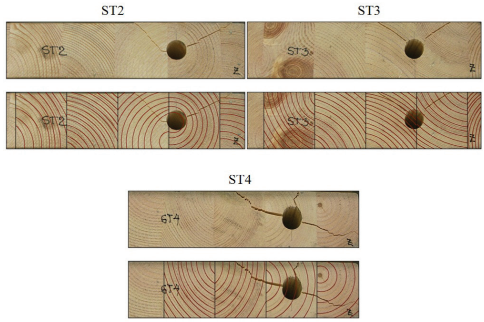

Fig. 7: View of the faces of specimens ST2, ST3 and ST4 with idealized pattern of annual rings in places where approach P2 was applied.

For the orthotropic material model (approach P1), the Tsai-Wu failure criterion was used for predicting failures of the wooden material, formulated as 2), which provides more conservative results than using the interactive Hoffman failure criterion (Pěnčík 2014). When using the approach P2), the maximum stress criterion in tension described in (Vinson and Sierakowski 2002) was used to identify brittle fracture of the wooden material in tension.

In the analyses, elastic and material constants for wood from Scots pine were used given in (Tab. 2). Elastic constants, i.e. the modulus of elasticity in the longitudinal direction E_L (direction L), modulus of elasticity in the tangential direction E_T (direction T), modulus of elasticity in the radial direction E_R (direction R), the shear modulus G_{LT}, G_{TR} and G_{LR} in planes LT, TR and LR and the Poisson coefficients v_{LT}, v_{TR} and v_{LR} in planes LT, TR and LR were taken from (Matovič 1981) and (Požgaj et al. 1997). These constants meet the criteria 3) (Gillis 1972), (Hallai 2008) and criteria 4) (Xavier 2007), which reflect their interdependence.

$$E_L > E_R > G_{LR} \approx G_{LT} > E_T > G_{RT} \tag{3}$$

$$E_L >> E_R > E_T, \quad G_{LR} > G_{LT} >> G_{RT}, \nu_{LR} > \nu_{LT} > \nu_{RT} \tag{4}$$

Tab. 2: Elastic and material constants of Scots pine (Pinus sylvestris); E, G, f (MPa), v(–).

E_L	E_R	E_T	G_{RT}	G_{LT}	G_{LR}	v_{RT}	v_{LT}	v_{LR}
14300	700	545	500	800	1230	0.38	0.04	0.03
f_{Lt}	f_{Lc}	f_{Tt}	f_{Tc}	f_{Rt}	f_{Rc}	f_{LR}	f_{LT}	f_{RT}
101.0	43.0	4.927	5.4	5.4	5.2	7.5	7.3	2.3

Material constants shown in (Tab. 2), where f_{Lt}, f_{Tt}, f_{Rt} are strengths in tension, f_{Lc}, f_{Tc}, f_{Rc} are strengths in compression in material directions L, T and R, f_{LR}, f_{LT}, f_{RT} are shear strengths in material planes, are based on the constants listed in (Matovič 1981) and (Požgaj et al. 1997) and were modified because of their use with an anisotropicplastic material model so that to meet the condition of plasticity incompressibility 5) and the condition 6) providing a closed area of plasticity in an elliptic cross section both in (2012), (Moses and Prion 2002).

$$\frac{f_{Lt} - f_{Lc}}{f_{Lt} f_{Lc}} + \frac{f_{Tt} - f_{Tc}}{f_{Tt} f_{Tc}} + \frac{f_{Rt} - f_{Rc}}{f_{Rt} f_{Rc}} = 0 \tag{5}$$

$$M_{11}^2 + M_{22}^2 + M_{33}^2 - 2\left(M_{11}M_{22} + M_{22}M_{33} + M_{11}M_{33}\right) < 0 \tag{6}$$

where: $M_{ii} = \dfrac{f_{Lt} f_{Lc}}{f_{it} f_{ic}}$ for $i = L, T, R$

Material behaviour of the spread footing and steel supporting device was described by an isotropic material model (E = 210 GPa, ν = 0.33, ρ = 7850 kg.m^{-3}). The isotropic material model was also used in modelling the behaviour of the rubber case (E = 10 MPa, ν = 0.475, ρ = 50 kg.m^{-3}, (2012b)). To describe the behaviour of the material of the steel support bars ⌀ 16 mm, bilinear isotropic material model without hardening in tension and compression was used (E = 210 GPa, σ_y = 235 MPa, ν = 0.33, ρ = 7850 kg.m^{-3}).

Boundary conditions for rubber cases and the steel supporting device were considered so that to model the real mounting of a stair on the loading track (Fig. 3). Boundary conditions for rubber cases were defined on their cylindrical surface in cylindrical coordinate system (index cyl) and did not allow rotating of the case in the wall. Movable support of a stair on the supporting device was modelled by preventing vertical displacement in its lower part (Fig. 6). The connection of a stair with the supporting device was modelled by using contact finite elements of the type TARGE170 and CONTA174, with consideration of standard behaviour of the contact. In case of separation of a stair from the supporting device, i.e. opening of the contact, zero normal stress was considered (σ = 0). For a contact between wood and steel, the value of friction coefficient was also considered μ = 0.2 according to (2004). Contact finite elements with the same behaviour were used in modelling the connection of the rubber case and the steel support bar. Also in this case, the friction coefficient of rubber and steel was considered according to (2004) to have the value μ = 0.75. Only in the upper part, the connection of steel support bars with stairs was considered in a sector of 120°.

The numerical models were loaded through the spread footing with surface loading, specified with the use of 3D finite elements of the type SURF154. With regard to the type of analysis, incremental loading was considered. The calculations were carried out in steps using a user macro specified in (Pěnčík 2013), created using the programming language APDL – ANSYS Parametric Design Language (2012a). With regard to the use of anisotropic plastic material model, the calculation was materially nonlinear. In the analysis, geometric nonlinearity was also taken into consideration, the use of which is conditional on the use of contact elements.

RESULTS AND DISCUSSION

In the numerical modelling, all test specimens ST2, ST3 and ST4 have failed in the location of a stair supporting onto a steel support bar ⌀ 16 mm, which is closer to the place where the load takes effect, or more precisely, close to the measuring point C (Fig. 3). An example of a failure in numerical analysis or more precisely, disrupted finite elements in numerical model ST4 is shown in (Fig. 8).

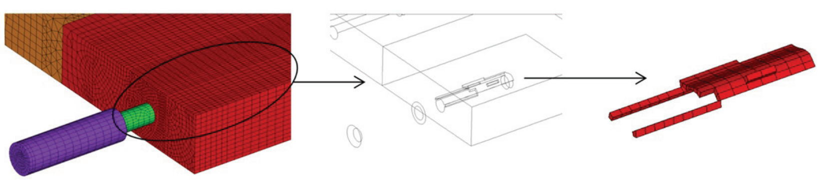

Fig. 8: Failure of analysis model ST4.

Values of the maximum force *F* at which the numerically modelled test specimens ST2, ST3 and ST4 have broken are shown in (Tab. 1). The difference between the experimentally and numerically identified values of the maximum force is in the case of the test specimen ST2 +10.62 % (6.98 kN/6.31 kN), ST3 –8.36 % (5.70 kN/6.22 kN) and ST4 +14.83 % (8.3 kN/9.78 kN). In the average, the test specimens subjected to experimental testing have broken under the force *F* = 7.00 kN. The average force identified by numerical modelling was 5.8 % higher and it had the value of *F* = 7.43 kN.

The graphic relationship between vertical displacement and the applied force for test specimens ST2, ST3 and ST4 subjected to experimental tests (measuring points *A* to *E*) and numerical analysis (points *Av* to *Ev*), while taking into account the same scales of coordinate axes, is shown in graphs in (Fig. 9). From loading curve diagrams it is evident that calculation models are more rigid than the test specimens.

The values of vertical displacement obtained for the measuring points in the experimental tests compared with the calculated values of vertical displacement in the measuring points are 20-30 % higher. In the measuring point *E* the experimentally identified values of vertical displacement (value $U_{Y,E}$ from (Tab. 1)) are higher than the values identified by numerical analysis. The difference is in the case of the specimen ST2 +26.47 %, ST3 +34.39 % and ST4 +23.43 %; the average difference is 28.10 %.

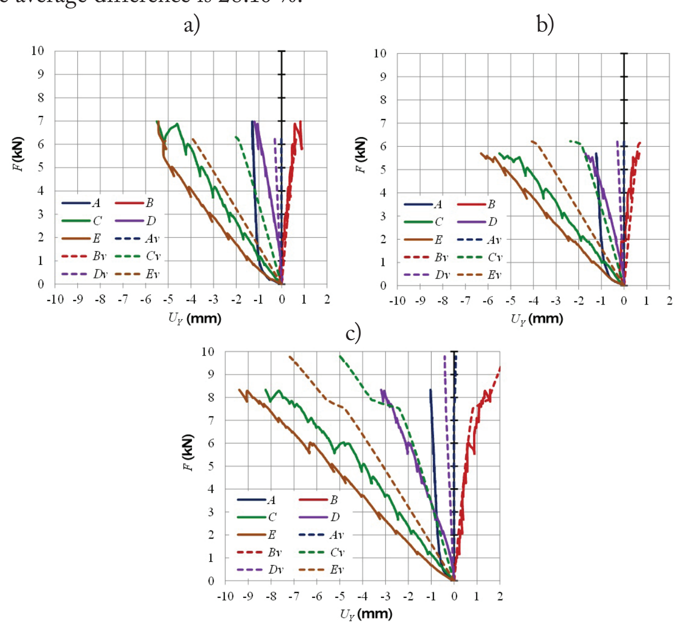

Fig. 9: Graph of relationship between vertical displacement UY (mm) in the measuring points and the applied force F (k) for specimens ST2 a), ST3 b) and ST4 c). A to E – experimental tests; Av to Ev – numerical analysis.

The loading curves for the measuring points *A* and *Av* indicate a different behaviour of the computational model and experimental tests. The loading curves obtained from experimental tests indicate that during the tests the supporting device depicted in (Fig. 3a) has probably moved vertically 1.14 mm in average (Fig. 9). The second possible explanation of this phenomenon, i.e.

deformation of the tested stairs by an impression was, with regard to an intact lower surface of all tested specimens in places of their mounting to the supporting device excluded. In the case of removal of this influence from the experimentally identified values of vertical displacement of the measuring point *E* (value $U_{Y,Em}$ from (Tab. 1)), the experimentally identified values of vertical displacement are higher than the values identified by numerical analysis. The difference is in the case of specimen ST2 +3.38 %, ST3 +20.54 % and ST4 +14.00 %; the average difference is 12.64 %.

A very good agreement in the pattern of loading curves was obtained for the measuring points *B* and *Bv* (Fig. 9), which is indicative of the suitability of using the modelling of the mounting of stairs on the supporting device using contact finite elements of the type TARGE170 and CONTA174. Using of contact elements for modelling of mounting of stairs as modelling of the connection of the rubber case and the steel support bar corresponds with conclusions in Pousette (2003) and Pousette (2006).

CONCLUSSIONS

Using a 3D numerical analysis of a wooden staircase with one-sided suspended stairs made from Scots pine place of increased concentration of stress was identified. To identify this place by wood modelling, a rectangular orthotropic material model was used and failures were predicted using the Hoffman and Tsai-Wu criteria. To assess the behaviour and bearing capacity of the identified place, i.e. place of supporting of stairs into the wall using steel support bars were experimentally tested using partial experimental models.

The identified detail was numerically analysed. The presented results of experimental tests and numerical analyses allowed to make their comparison. The results confirm the possibility of replacing the time-consuming and expensive experimental tests by numerical modelling, using appropriate material models and prediction of material failure. The results also document the possibility of combining different material models within one numerical model, and different criteria. In this case was combined an orthotropic material model without considering the curvature of annual rings, i.e. rectangular orthotropy, with anisotropic plasticity material model taking into account the curvature of annual rings. The more accurate anisotropic plasticity material model was used for places where creation of failures was expected. To predict failures in the wood material using the rectangular orthotropic material model, interactive failure criteria were applied. Brittle failures of wood in tension described by the anisotropic plasticity material model were identified using a non-interactive failure criterion.

In view of the findings that experimental testing can be replaced with numerical modelling using appropriate material models and prediction of material failure, modification of a wooden staircase with one-sided suspended stairs was proposed using subsequent numerical analyses. Adjustments were designed to improve the versatility and applicability of prefabricated staircases and improve the versatility of the constructional system with a reduction in production costs of staircases by material savings. As part of the adjustment it was designed to reduce the number of steel rods from 24 pieces (Fig. 2) to 4 pieces for the entire staircase. Another adjustment was modification of the thickness of the stairs from the original thickness of 50 mm to 40 mm. The solution of supporting the stairs into the wall using steel support bars was replaced using steel rolled profiles of L shape (Pěnčík 2013) so that the material of stairs is not weakened by holes which according to the experimental tests and numerical analyses had a great influence to its bearing capacity. The proposed changes had a positive effect on the size of the maximum force needed to damage this modified detail F = 15.80 kN, which was by 124.3 % higher compared with the original average force needed for the failure F = 7.00 kN, while at the same time decreasing the thickness of stairs from the original thickness of 50 mm to 40 mm.

The designed modifications which are in detail described in (Pěnčík 2013) were incorporated into two prototypes of direct prefabricated staircase with one-sided suspended stairs that have been experimentally tested in 1:1 scale in the laboratory of Institute of Building Testing, Faculty of Civil Engineering, Brno University of Technology. Description and evaluation of this static load tests is given in Pěnčík (2013) and Pěnčík and Lavický (2011). The modifications were verified in practice.

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REFERENCES

1. ANSYS, 2011: ANSYS® Academic Research, Release 14.0. Southpointe, USA: ANSYS, Inc.
2. Bergman, R., Cai, Z., Carll, Ch.G., Clausen, C.A., Dietenberger, M.A., Falk, R.H., Frihart, Ch.R., Glass, S.V., Hunt, Ch.G., Ibach, R.E., Kretschmann, D.E., Rammer, R.J., Ross, R.J., Stark, N.M., 2010: Wood handbook, Wood as an engineering material (All Chapters). Forest Products Laboratory. Wood handbook – Wood as an engineering material. General Technical Report FPL-GTR-190. Madison, WI: U.S. Department of Agriculture, Forest Service, Forest Products Laboratory.
3. Bodig, J., Goodman, J.R., 1973: Prediction of elastic parameters for wood. Wood Science 5(4): 249-264.
4. Bucur, V., 2006: Acoustics of wood. 2nd ed. New York: Springer, 394 pp.
5. Camanho, P.P., 2002: Failure criteria for fibre-reinforced polymer composites. Failure criteria for fibre-reinforced polymer composites (online). No. 1 (Accessed March 15, 2014). http://paginas.fe.up.pt/~stpinho/teaching/feup/y0506/fcriteria.pdf.
6. ČSN 73 4130, 2010: Stairways and slidingramps – Basic requirements. (Schodiště a šikmé rampy – Základní požadavky) (in Czech).
7. ČSN EN 1991-1-1, 730035, 2004: Eurocode1: Actions on structures – Part 1-1: General actions – Densities, self-weight, imposed loads for buildings. (Eurokód 1: Zatížení konstrukcí - Část 1-1: Obecná zatížení – Objemové tíhy, vlastní tíha a užitná zatížení pozemních staveb) (in Czech).
8. Danielsson, H., Gustafsson P.J., 2013: A three dimensional plasticity model for perpendicular to grain cohesive fracture in wood. Engineering Fracture Mechanics (98): 137-152.
9. Dinckal, C., 2011: Analysis of elastic anisotrophy of wood material for engineering application. Journal of Innovative Research in Engineering and Science 2(2): 67-80.
10. Gillis, P. P., 1972: Orthotropic elastic constants of wood. Wood Science and Technology 6(2): 138-156.

11. Hallai, J., 2008: Fracture of orthotropic materials under mixed mode loading: EM 388F Fracture Mechanics: Term Paper. Austin, Texas: University of Texas at Austin, Department of Aerospace Engineering and Engineering Mechanics, (online). (Accessed March 20, 2014) http://imechanica.org/files/Julian's%20Term%20Paper.pdf.
12. Hoffman, O., 1967: The brittle strength of orthotropic materials. Journal of Composite Materials 1: 200-206.
13. JEMA Svitavy a.s., 2006: Wooden stair cases: Product katalog. (Dřevěná schodiště: Produktový katalog) (in Czech).
14. Mascia, N.T., Lahr, F.A.R., 2006: Remarks on orthotropic elastic models applied to wood. Material Research 9(3): 301-310.
15. Matovič, A., 1981: Wood science. (Nauka o dřevě). 2. Brno: Vysoká škola zemědělská v Brně, 159 pp (in Czech).
16. Moses, D.M., Prion, H.G.L., 2002: Anisotropic plasticity and failure prediction in wood composites. In: Ansys.net (online). 2002 (Accessed March 14, 2014). http://ansys.net/ansys/papers/nonlinear/anisotropic_plasticity_failure_prediction_wood.pdf.
17. Moses, D.M., Prion, H.G.L., 2004: Stress and failure analysis of wood composites: A new model. Composites: Part B: Engineering (35): 251-261.
18. Pěnčík, J., 2014: Modelling of wood using an orthotropic material model with failure criteria. (Modelování dřeva pomocí ortotropního materiálového modelu s kritérii porušení). Stavební obzor (1/2): 4-10 (in Czech).
19. Pěnčík, J., 2013: Wooden prefabricated staircase with one-sided suspended stairs. (Dřevěné segmentové schodiště s jednostranně zavěšenými stupni). Brno, Habilitační práce. VUT v Brně, 185 pp (in Czech).
20. Pěnčík, J., Lavický, M., 2011: Combination of methods of mathematical modelling and experimental tests for the design of modern staircases. (Matematické modelování a experimentální testy pro navrhování moderních schodišť). Časopis Stavebnictví (9): 42-47 (in Czech).
21. Pěnčík, J., Lavický, M., Havířová, Z., 2013: Using of method of numerical modelling for identification of critical places of prefabricated staircase with one-sided suspended stairs. Využití metody numerického modelování při identifikaci kritických mist segmentovéhos chodiště s jednostranně zavěšenými stupni Stavební partner. E-magazín II(3): 1-4.
22. Pěnčík, J., Lavický, M., Schmid, P., Daněk, P., Žítt, P., 2006: Tests of details of straight prefabricated staircase with one-sided suspended stairs: Research report HS 12625002. (Zkoušky detail přímého třmenové hoschodiště: Zpráva HS 12625002). Brno: VUT v Brně, Fakulta stavební, Ústav stavební mechaniky, 60 pp (in Czech).
23. Pousette, A., 2003: Full-scale test and fi nite element analysis of a wooden spiral staircase. Holz als Roh-und Werkstoff 61(1): 1-7.
24. Pousette, A., 2006: Testing and modeling of the behavior of wooden stairs and stair joints. Journal of Wood Science 52(4): 358-362.
25. Požgaj, A.; Chovanec, D.; Kurjatko, S.; Babiak, M., 1997: Structure and wood properties. (Štruktúra a vlastnosti dreva). Priroda, Bratislava, 485 pp (in Slovak).
26. Schellekens, J.C.J.R. De Borst, R., 1990: The use of the Hoffman yield criterion in finite element analysis of anisotropic composites. Computers & Structures 37(6): 1087-1096.
27. Tsai, S.W., Wu, E.M., 1971: A general theory of strength for anisotropic materials. Journal of Composite Materials (5): 58-79.
28. Veselý, J., Mikš, L., 2010: Technical requirements for construction: Part 6, Stairs and ramps. (Technické požadavky na výstavbu: Díl 6, Schodiště a rampy, Část 7, Díl 6, Kapitola 1). Praha: Verlag Dashöfer, nakladatelství, spol. s r. o., 22 pp (in Czech).

29. Vinson, J.R., Sierakowski, R., 2002: The behavior of structures composed of composite materials. 2nd ed. Boston: Kluwer Academic Publishers, 435 pp.
30. Xavier, J., 2007: Characterisation of the wood stiffness variability within the stem by the virtual fields method: Application to *P. pinaster* in the LR plane. (Identification de la variabilité des rigidités du bois á l'intérieur de l'arbrepar la méthode des champs virtuels: Application au *P. Pinaster* dans le plan LR. Paris, 2007. Thése pour obtenir la grade de Docteur. E.N.S.A.M (Ecole Nationale Supérieure des Arts etMétiers). (online). (Accessed March 21, 2014) (in French).
31. 2004: Coefficient for static friction of steel chart. Carbide depot carbide inserts and cutting tools (online). (Accessed March 20, 2014). http://www.carbidedepot.com/formulas-frictioncoefficient.htm.
32. 2006: "Spider8 from HBM" (online), HBM Inc.,(Accessed March 20, 2014). http://www.hbm.com/fileadmin/mediapool/hbmdoc/technical/b0409.pdf. First published 2006.
33. 2009: ANSYS Element Reference: Release 12.0 (online). Southpointe, USA: ANSYS, Inc., 1690 pp. (Accessed March 18, 2014).
34. 2012:ANSYS Mechanical APDL Theory Reference: Release 14.5 (online). Southpointe, USA: ANSYS, Inc., 298 pp. (Accessed March 20, 2014).
35. 2012a: ANSYS Parametric Design Language Guide: Release 14.5 (online). Southpointe, USA: ANSYS, Inc., 108 pp. (Accessed March 20, 2014).
36. 2012b: Elastic properties and young modulus for some materials. The engineering toolbox (online). (Accessed March 19, 2014). http://www.engineeringtoolbox.com/young-modulus-d_417.html.
37. 2012c: "HBM WA T – Inductive displacement transducer (Probe)" (online), HBM Inc., (Accessed March 20, 2014). http://www.hbm.com/en/menu/products/transducers-sensors/displacement/wa-t/#c71014.

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