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CALCULATION OF CORRUGATED BOARD FLAT CRUSH RESISTANCE

Zbigniew Kołakowski
Lodz University of Technology, Faculty of Mechanical Engineering, Department of Material Strength
Lodz, Poland

Włodzimierz Szewczyk, Krzysztof Głowacki
Lodz University of Technology
Institute of Papermaking and Printing
Lodz, Poland

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ABSTRACT

The article discusses an effect of Young's modulus of material for corrugated medium, flute geometric parameters and humidity on corrugated board flat crush resistance. On the basis of measurement results, mathematical relationships were developed to calculate values of the flat crush test (*FCT*) based on Young's modulus in machine direction and corrugated medium thickness as well as flute height and pitch.

KEYWORDS: Corrugated board, flat crush resistance, humidity.

INTRODUCTION

Properties of corrugated board used as a packaging material differ significantly from properties of wood, despite the fact that both materials are made of the same fibres. Due to a spatial structure owing to corrugated medium, corrugated board has significantly lower specific weight and better cushioning properties than wood. However board is more sensitive to humidity effect than wood. One of the parameters used to evaluate corrugated board usability is the *FCT* determining the highest pressure applied on the external flat layer of the board which can be transferred until a flute is crushed. Fig. 1 shows the phases of flute crushing when using the *FCT*.

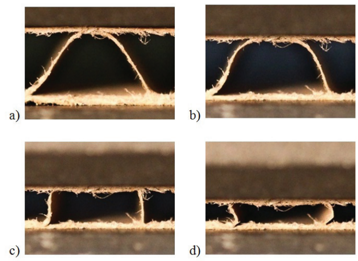

Fig. 1: Flute crushing phases.

Despite the fact that many works discuss testing flat crush resistance Krusper et al. 2007, Jiménez-Caballero et al. 2009) there are very little information on impact of humidity on *FCT*. Simple, based on the material strength theory, the method for calculation of this parameter on the basis of properties of corrugated medium has not been developed yet and this is the aim of this article.

MATERIAL AND METHODS

During the crushing test, the following phases of flute deformation can be observed
– original shape, load is not applied (Fig. 1a),
– tip flattening and strengthening of flute flanks (Fig. 1b),
– arranging flute flanks in perpendicular direction to liners (Fig. 1c),
– collapse of flute flanks, flute failure (Fig. 1d).

Assuming, that flute shape with cross-section parameters shown in Fig. 2a at the moment of compression failing will have a form shown in Fig. 2b, flute height at the moment of collapse h_1 can be calculated on the basis of the relationship (Eq. 1).

$$h_1 = \frac{t(\lambda - 1)}{2} \tag{1}$$

where: t – flute pitch,
λ – corrugating factor (length of corrugated layer before corrugating in relation to its length after corrugating).

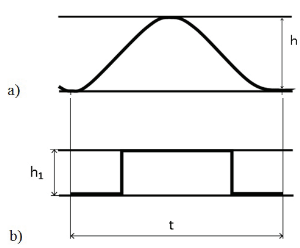

Fig. 2: Change in flute shape.

Taking into consideration the fact that at the final phase the flute flanks are arranged perpendicularly to the surface of linerboard, it can be assumed that the *FCT* values depend on resistance of corrugated medium to crush with forces applied to its surface in machine direction as carried out in article (Kołakowski et al. 2014). In many cases, correlation formulas based on this assumption allow to obtain values calculated theoretically which are close to real values. However, they oversimplify mechanism of flute failure as they do not consider buckling.

A model assuming that the flat crush resistance for corrugated board can be calculated treating flanks of corrugated medium as compressed posts placed perpendicularly to linerboard is more similar to real loads applied to a flute in the *FCT*. With such assumption, according to the articles (Rhanicyn 1955, Jennings 2004, Bazant and Cedolini 2003), and with assumption that beam stiffnes will be calculated from Eq. 2.

$$\beta = \frac{Q}{\delta} = \frac{\alpha \cdot E_{MD} \cdot L \cdot g^3}{12 h_1^3} \tag{2}$$

where: α – coefficient for beam elastic fixing,
δ – beam deflection,
β – beam stiffness,
Q – force ($\delta\beta$),
E_{MD} – Young's modulus for corrugated medium in machine direction,
L – combined length of flute flanks on the surface of a crushed sample of corrugated board,
g – thickness of corrugated medium,

adequate for cantilever beam of length h_1 elastically fixed, loaded with transversal force Q, critical force P_{kr} in flute flanks of corrugated board at the moment of their failure can be calculated as a force corresponding with buckling of a bar supported on both ends on elastic support stands from Eq. 3

$$P_{kr} = \frac{\beta \cdot h_1}{2} = \frac{\alpha \cdot E_{MD} \cdot L \cdot g^3}{24(\mu h_1)^2} \tag{3}$$

where: μ – coefficient of buckling height.

Using Eq. 3, critical stress σ_{kr} can be determined in flute flanks at the moment of their buckling from Eq. 4

$$\sigma_{kr} = \frac{P_{kr}}{L \cdot g} = \frac{\alpha \cdot E_{MD} \cdot g^2}{24(\mu h_1)^2} \tag{4}$$

Proportion between pressure applied at the moment of flute failure on the surface of the *FCT* sample, and critical stress σ_{kr} is the same as proportion between the flute pitch t and its thickness g. Using this equation, the flat crash resistance for corrugated board can be defined with Eq. 5

$$FCT = \frac{\alpha}{24 \cdot \mu^2} \frac{E_{MD} \cdot g^3}{h_1^2 \cdot t} = \gamma \cdot \frac{E_{MD} \cdot g^3}{h_1^2 \cdot t} \tag{5}$$

After rearrangement the relationship can be expressed as

$$FCT = \frac{\alpha}{24 \cdot \mu^2} \frac{E_{MD} \cdot g^3}{h_1^2 \cdot t} = \gamma \cdot \frac{E_{MD} \cdot g^3}{h_1^2 \cdot t} \tag{6}$$

where: $\gamma = \frac{\alpha}{24 \cdot \mu^2}$

Owing to Eq. (6) it is possible to replace coefficient of fixing elasticity α and coefficient of buckling height μ with one coefficient γ.

Value of coefficient γ can be calculated from Eq. 7, obtained by rearrangement Eq. 6

$$\gamma = \frac{FCT}{E_{MD}} \cdot \frac{h_1^2 \cdot t}{g^3} \tag{7}$$

Assuming that an expression containing combination of geometric parameters is expressed as η

$$\eta = \frac{h_1^2 \cdot t}{g^3} \tag{8}$$

coefficient γ can be described with relationship

$$\gamma = \frac{FCT}{E_{MD}} \cdot \eta \tag{9}$$

As it is well known (Głowacki and Szewczyk 2011, Olejnik and Wysocka-Robak 2005, Roisum 1993, Schröeder and Bensarsa 2002, Zauscher et al. 1996, 1997), properties of paper and paper products depend on humidity which, for practical reasons, is often described as equilibrium humidity obtained by fibrous material as a result of conditioning in air with defined parameters – relative humidity *RH* and temperature. Coefficient γ depends therefore from geometric parameters of board cross-section and mechanical properties of corrugated medium which depend from paper humidity. Geometric parameters of flute cross-section do not depend from humidity thus an effect of humidity can be described by changes in values of quotient FCT/E_{MD} in humidity function. Coefficient γ depends on flute pitch and height and corrugating coefficient. All these parameters affect value of flute height at the moment of collapse h_1, as it is described by Eq. (1) hence changes in coefficient γ caused by changes in geometric parameters of flute cross-section can be analyzed in the function of height h_1. Another method for description of these changes is their presentation in the function of parameter η.

Six grades of corrugated board were tested. Their properties are listed in Tab. 1.

Tab. 1: Geometric parameters of corrugated board used in the tests.

Corrugated board	Flute type	t (mm)	h (mm)	λ	g (mm)
T1	C	7.9	3.43	1.38	0.15
T2	B	6.5	2.43	1.30	0.15
T3	B	6.2	2.63	1.32	0.17
T4	C	7.5	3.47	1.399	0.18
T5	B	6.5	2.29	1.268	0.18
T6	B	6.5	2.29	1.268	0.18

The measurements of the *FCT* and Young's modulus for corrugated medium in machine direction EMD, after air-conditioning of samples at temperature of 23°C and at different humidity were carried out for all the corrugated board grades.

RESULTS AND DISCUSSION

The measurement results are shown in Figs. 3 and 4.

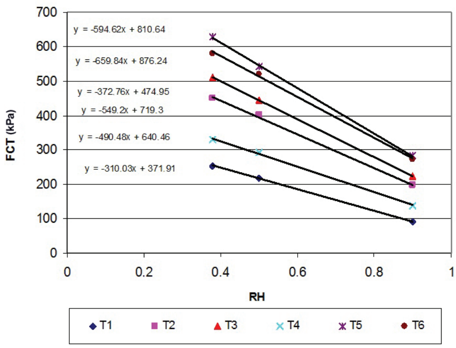

Fig. 3: FCT in function of air humidity RH at which corrugated board was conditioned.

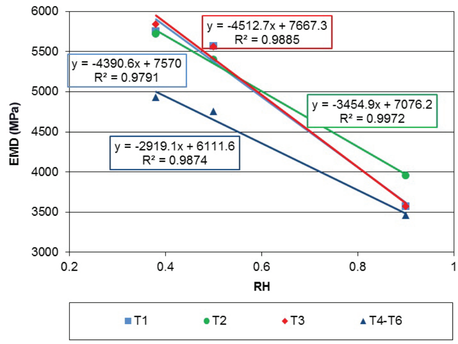

Fig. 4: EMD for corrugated medium in function of air humidity RH.

As it can be seen in the tested range of humidity, changes in the *FCT* and E_{MD} can be described by a linear relationship and consequently the value of quotient FCT/E_{MD} and therefore the value of coefficient γ in function of humidity *RH* also can be described by the linear relationship, as it is shown in Fig. 5.

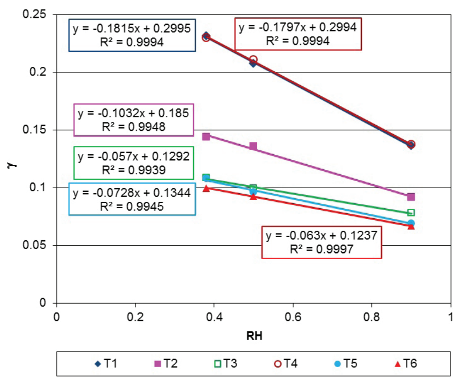

Fig. 5: Coefficient γ in function of air humidity RH at which board was conditioned.

Figs. 6 and 7 show changes in values of coefficient *a* and coefficient *b* of a linear relationship describing changes in coefficient γ

$$\gamma = a \cdot RH + b \quad (10)$$

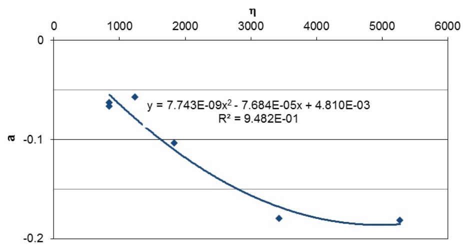

Fig. 6: Change of coefficient a depending on parameter η.

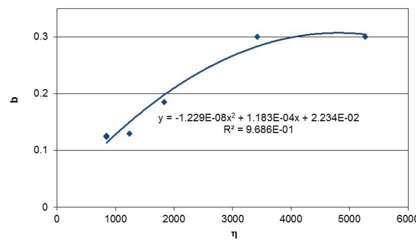

Fig . 7: Change of coefficient b depending on parameter η.

Figs. 8 and 9 show changes of coefficient a and b of relationship (10), determined in function of height h_1.

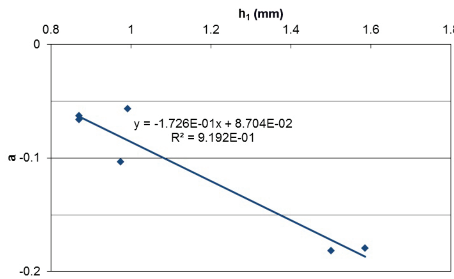

Fig. 8: Change of coefficient a depending on h_1.

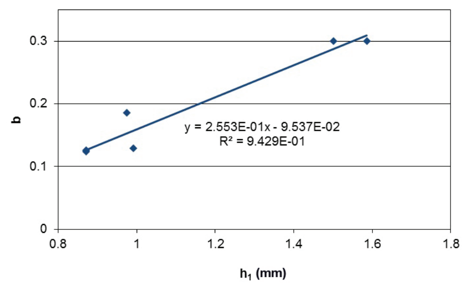

Fig. 9: Change of coefficient b depending on h_1.

Using equations describing changes of parameter a and parameter b with formula (10) a relationship between γ and RH can be determined, and therefore knowing Young's modulus for corrugated medium in machine direction and flute geometric parameters, the *FCT* for corrugated board at defined humidity can be calculated with relationship (6).

Such calculation was done determining parameter γ in two ways: on the basis of correlation with parameter η (Figs. 6, 7), and the correlation with h_1 (Figs. 8, 9). Comparison of measurement and calculation results is shown in Tab. 10.

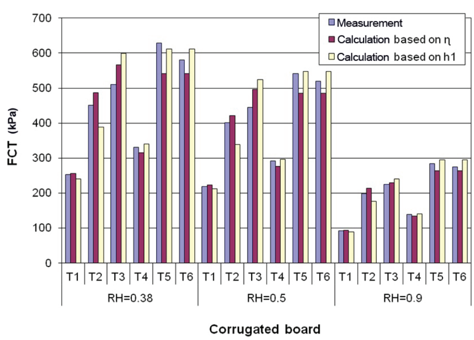

Fig. 10: Comparison of measured and calculated FCT values.

As Fig. 10 shows, both methods for theoretical calculation of the *FCT* values allow to have good results. In both methods, for all the tested board grades and humidity levels, on average the difference between the measured and calculated values is lower by 7 % from the measured value. The average discrepancies between the values for the *FCT* measurement and calculation for each humidity level are similar and they range from 4.1 to 7.8 %.

In case of the method based on determination of coefficient γ on the basis of value h_1, the highest difference between real and calculated values was 17.9 and 14 % in case of the method based on determination of coefficient γ on the bases of value η.

When compared to the methods described by specialized literature (Krusper et al. 2007, Jiménez-Caballero et al. 2009), the methods for theoretical calculation of *FCT*, presented in the article, do not require application of numerical methods and they are easier-to-use. Additionally, they enable to calculate corrugated board flat crush resistance after conditioning in air of different humidity.

CONCLUSIONS

Both presented methods for calculation of flat crush resistance for corrugated board give good results in tested range of board humidity, obtained as a result of drying equilibrium with air. However, the method based on correlation between coefficient γ and height h_1 is slightly easier to use. Due to the fact that small number of corrugated board grades was tested, the presented dependence of parameter a and parameter b from coefficient η and height h_1 should be treated as hypothetical.

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Zbigniew Kołakowski*
Lodz University of Technology
Faculty of Mechanical Engineering
Department of Material Strength
Żeromskiego 116
90-924 Lodz
Poland
Corresponding author: zbigniew.kolakowski@p.lodz.pl

Włodzimierz Szewczyk, Krzysztof Głowacki
Lodz University of Technology
Institute of Papermaking And Printing
Żeromskiego 116
90-924 Lodz
Poland