

**GAMMA PROCESS MODEL FOR TIMBER-CONCRETE  
COMPOSITE BEAM DETERIORATION PREDICTION**

TATYANA MICIC  
CITY UNIVERSITY LONDON  
LONDON, UNITED KINGDOM

DRAGOSLAV STOJIC  
UNIVERSITY OF NIŠ, FACULTY OF CIVIL ENGINEERING AND ARCHITECTURE  
NIŠ, SERBIA

MIOMIR STANKOVIC  
UNIVERSITY OF NIŠ, FACULTY OF OCCUPATIONAL SAFETY  
NIŠ, SERBIA

NIKOLA VELIMIROVIC  
UNIVERSITY OF NIŠ, FACULTY OF CIVIL ENGINEERING AND  
ARCHITECTURE  
UNIVERSITY OF NIŠ, FACULTY OF SCIENCE AND MATHEMATICS  
NIŠ, SERBIA

(RECEIVED DECEMBER 2015)

**ABSTRACT**

The main goal of maintenance management is to accurately predict the performance of structures over their life-cycle in order to develop the optimal maintenance programs. The aim of this paper is to present one of the prediction models of aging of timber concrete composite structures which will capture the true nature of deterioration. We focus on modelling the deterioration of deflection in the mid-span of the timber-concrete composite beam under a service load. Due to the nature of this composite beam, relative mid-span deflection is generally uncertain and non-decreasing over time, so it could be regarded as a gamma process. The progress of deterioration and estimate its service life will be presented.

**KEYWORDS:** Deterioration model, gamma process, timber-concrete composite.

## INTRODUCTION

When designing structures and infrastructures it is crucial to consider that they are constructed and maintained in the most optimal way so that they could be safely used during the expected service life. The environmental conditions are the main cause for the deterioration and the reduction in safety and reliability of the existing structures and infrastructures. The main goal is to accurately predict the performance of these structures over their life-cycle in order to develop the optimal maintenance programs. The aim of this paper is to present one of the prediction models of aging of timber concrete composite structures which will capture the true nature of deterioration.

The timber-concrete composite (TCC) structure is a structural system in which a timber beam is connected to an upper concrete flange using different types of connectors. As other composite structural systems the timber concrete composite is designed to take advantage of the compatible features of both materials used in the composite. The best features of both materials can be exploited because bending and tensile forces induced by gravity loads are resisted primarily by the timber beam and compression by the concrete slab, while the connection system transmits the shear forces between the two components. The stiffness of the section is significantly increased by connecting a concrete flange to a timber beam rather than when the timber section is used alone. Due to its lower weight compared to the reinforced concrete section, TCC has reduced seismic effects. The connection between the constituent materials is the most important part of composite structure. A global collapse of the beam can happen due to a failure of the shear connector thus, the shear connector needs to be stiff, further more it needs to have a certain strength or shear capacity. Yeoh et al. (2011) shows a survey on the state of the art of timber concrete composite research in the recent years. TCC structural systems are successfully used in bridges, piers, platforms and in upgrading and strengthening existing timber floors in residential and office buildings.

In order to properly predict the long term behaviour of TCC, we need to consider the behaviour of the constituent materials. Component materials will deteriorate at different pace over the life cycle. While concrete displays creep and shrinkage effects, timber behaves in a more complex way due to its sensitivity to the environmental effects, which causes the changes of the moisture content in timber and can accelerate the deformations and reductions in strength. For instance an increase in relative humidity leads to timber moistening with swelling and an overall increase in deflection of the beam. In the same manner, a decrease in temperature produces a larger shrinkage of concrete slab compared to timber beam, with an overall increase in deflection. It has been shown in published records of the long-term loading experiments that the connections can creep even more than timber.

## MATERIAL AND METHODS

### **Deterioration modelling**

Deterioration prediction plays a major part in the efficient management of structures in terms of maintenance, repair and possible replacement. That is why we use the deterioration model to approximate and predict the actual process of ageing, in terms of safety and reliability of structures. The existing deterioration models which have been discussed in literature over the years can be generally classified into two categories as deterministic and probabilistic models.

Deterministic deterioration models are broadly accepted in the maintenance management process. Any analytical model designed to estimate the long term behaviour of the timber-concrete composite sections has to include such diverse effects that develop at different time in the life cycle such as creep, shrinkage, mechano-sorptive creep, swelling etc. However, due to complexity of the phenomena involved, deterministic prediction models will have limited validity in real conditions.

Application of the probabilistic models is required in order to deal with the uncertainty in the engineering models and randomness in the influence of the environmental conditions. According to Frangopol et al. (2004) the probabilistic deterioration models can be generally classified into two categories: Random variable models and stochastic process models. The main idea of the random variable models is that one or more of the variables in deterioration model is random variable with certain probability distribution. We can distinguish three different random variable models: failure rate model, classical reliability index model and time-dependent reliability index models. It has been determined that random variable models have certain limitations and they are unable to capture temporal effects that could be relevant for long lifecycles such as those in constructions. These issues are even more important in composite sections in regards to various behaviour of component materials over the lifecycle. Therefore, it is more appropriate to base the deterioration modelling of civil structures and infrastructure in terms of a time-dependent stochastic process which has emerged as an alternative to random variable models in the late 1990s.

Deterioration is usually assumed to be a Markov process, stochastic process with independent increments. Types of Markov processes which are used for modelling stochastic deterioration are discrete time Markov process (Markov chains) and continuous time Markov process such as the Brownian motion with drift and the gamma process. The main difference between these two processes is that the first one has the independent increments and decrements and the second one has only independent increments which makes it more adequate for modelling deterioration that is monotone process.

### Gamma process modelling

Gamma process is a stochastic process with independent non-negative increments that are gamma distributed with an identical scale parameter. It was first proposed by Abdel-Hameed (1975) that the gamma process is suitable for modelling the deterioration randomly occurring in time. The gamma process presents the proper model for gradual damage monotonically accumulating over time, such as wear, fatigue, creep, crack growth, erosion, corrosion, swell etc (van Noortwijk 2009). In addition to this, the gamma process is considered as an appropriate approach for building element deterioration prediction (Edirisinghe et al. 2013) as well as for bridge deterioration prediction (Aboura et al. 2009). Gamma process can be defined as follows. The random quantity  $X$  that has a gamma distribution with shape parameter  $k > 0$  and scale parameter  $\theta > 0$  and its probability density function is given by:

$$Ga(x|k,\theta) = \frac{1}{\Gamma(k) \cdot \theta^k} x^{k-1} \exp\left\{\frac{-x}{\theta}\right\} \quad (1)$$

where:

$$\Gamma(a) = \int_{z=0}^{\infty} z^{a-1} e^{-z} dz \quad (2)$$

is the gamma function for  $a > 0$ . Also,  $k_{(t)}$  is assumed to be a non-decreasing, right-continuous, real-valued function for  $t \geq 0$ , with  $k(0) \equiv 0$ . We consider the gamma process with shape function  $k(t) > 0$  and scale parameter  $\theta > 0$ , as continuous-time stochastic process  $\{X(t); t \geq 0\}$  with the following properties:

- $X(0) = 0$  with probability one;
- $\Delta X(t) = X(t + \Delta t) - X(t) \sim Ga(\Delta k(t), \theta)$ ;  $\Delta k(t) = k(t + \Delta t) - k(t)$
- $\Delta X(t)$  are independent.

**Time-dependent timber-concrete composite beam deterioration**

It is crucial to obtain enough information about the state of the structure in order for it to be successfully maintained. Hence, the progress of deterioration of the observed structure and estimate of its service life needs to be quantified.

This paper will consider a simply supported TCC beam of span 4.2 m, with shear connectors made of glued steel bars. Concrete flange is made of normal-weight concrete C25/30 and solid timber beam is made of Spruce. Based on modern design codes such as Eurocode 5, both ultimate and serviceability limit states have to be satisfied. According to Fragiaccomo and Ceccotti (2004), for medium and long-span composite beams as well as composite beams exposed to external influences (i.e. bridges or roof structures), the most serious design criterion is the limit state of maximum deflection. This is the main reason why we consider modelling of deterioration of mid-span deflection of the TCC beam over time under a service load. In agreement with the Eurocode 5, for long-term deflections of simply supported beams where  $l$  is the span length,  $u_L$ , suggested limit value is  $l/250$ . As a result, when deflection reaches that assumed value, considered TCC beam will reach serviceability limit state. We recognize the relative mid-span deflection of the beam over time, which is defined as follows:

$$X(t) = \frac{u(t) - u_{el}}{u_{el}} \tag{3}$$

Elastic deflection  $u_{el}$  measured immediately after applying the service load is assumed to be an initial deflection, deflection at time  $t_0$ . After defining relative mid-span deflection over time, we can establish its serviceability limit value as  $\rho$ :

$$\rho = \frac{u_L - u_{el}}{u_{el}} \tag{4}$$

Due to the nature of timber-concrete composite beam, relative mid-span deflection is generally uncertain and non-decreasing over time, so it could be regarded as a gamma process (Fig. 1).

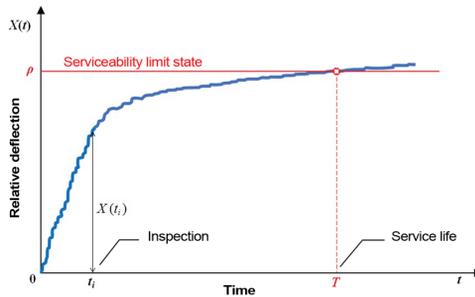


Fig. 1. Trend of the expected relative mid-span deflection increasing over time under service load.

### Estimation of the gamma process parameters

Modelling deterioration of the structure as a stochastic gamma process represents a combination of physical and statistical models. Deterioration prediction is conducted based on the analysis of the physical deterioration in accordance with engineering knowledge and experience and also based on the statistical data obtained from the condition inspections that reveal the current state of the deterioration. Assuming that a typical data set consists of inspection times  $t_i, i = 1, \dots, n$ , where  $0 = t_0 < t_1 < \dots < t_n$ , and corresponding observations of the cumulative amounts of deterioration  $x_i, i = 1, \dots, n$ , where  $0 = x_0 \leq x_1 \leq \dots \leq x_n$ . We can determine that the proposed model depends on the certainty of the visual inspection data as well as that the condition judgement depends on the ability and competence of the condition auditor.

In modelling the temporal variability in the deterioration, the key input is the trend of the expected deterioration of relative mid-span deflection of timber-concrete composite beam increasing over time. Empirical studies show that the expected deterioration at time  $t$  can often be represented as a power law:

$$E(X(t)) = k(t) \cdot \theta = ct^b \cdot \theta = at^b \propto t^b \quad (5)$$

for some physical constants  $c > 0, b > 0$  and  $\theta > 0$ . The gamma process is called stationary if the expected deterioration is linear in time, i.e., when  $b = 1$  and non-stationary when  $b \neq 1$ . There are well known engineering facts about the shape of the expected deterioration in terms of the parameter  $b$  and some of them are listed by van Noortwijk (2009). However, in case that there is no available data of the expected deterioration, as in our case, estimation of the parameter  $b$  can be performed based on the least square method that is suggested also by Nicolai et al. (2007). We conducted a parametric study with a number of TCC beams. Our initial approach was to simulate the condition of the beam at time  $t$  based on the accelerated degradation test (Haowei et al. 2015) using available deterministic model presented by Fragiaco (2006). Initially we generated accelerated degradation data as inspection data to simulate the condition of the beam that is monitored through periodic inspections and in that way inspections reveal the progress of the deterioration. The logarithm of obtained data were least-squares fitted using the number of logarithms of the power law functions allowing us to determine the average value of the parameter  $b$ . Using the normal equations for the linear functions we can obtain the system of equations which represents the system of normal equations for power law function (Fig. 2):

$$b \sum_{i=1}^n (\log t_i)^2 + \log c \sum_{i=1}^n \log t_i = \sum_{i=1}^n \log t_i \log x_i \quad (6)$$

$$b \sum_{i=1}^n \log t_i + n \log c = \sum_{i=1}^n \log x_i$$

By solving this system of equations we can obtain the value of the parameter  $b$ .

Long-term loading tests presented by Jorge et al. (2010) show the most significant increase in deflection during the first couple of years, this is especially true for composite beams exposed to outdoor conditions. The outcome of the conducted experiments revealed that concrete develops about 90 % of its end creep strain, while at the same time timber develops between 50 and 60 % of its end creep within the first 3 to 7 years.

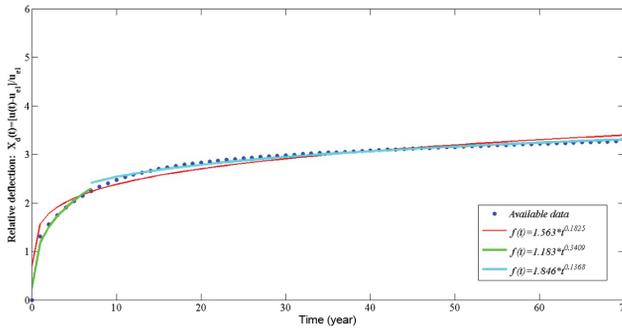


Fig. 2: Comparative least-square fitting of the available data by power law functions.

According to that, we decided to least squares fit the available data of up to year 7 and after year 7 using different power functions. This approach has provided significantly better results rather than when we least squares fitted all available data by using the same power function. The sum of squares due to error of the fit (SSE) is almost 10 times lower. A similar approach has been presented by Velimirović et al. (2013).

Tab. 1: Estimated parameter "b" according to different available data.

Inspection data (time period)	0-70	0-7	7-70
Estimated parameter b	0.1825	0.3409	0.1368

Beside the drastic increase of deflection in the first couple of years, during the greater part of their life cycle is relatively slow. Therefore, as a relevant value of parameter b, we will take the value obtained for the analyzed period from year 7 to year 70, which is 0.1368.

Having estimated the parameter b, it is essential to estimate the shape and scale parameters, the other two parameters of the gamma process. Maximum Likelihood Method is a common method of parameter estimation and it has been applied here. By using the Maximum Likelihood Method we can estimate parameters c and θ, that is done by maximizing the logarithm of the likelihood function of the observed deterioration increments, δ<sub>i</sub> = x<sub>i</sub> - x<sub>i-1</sub>, i = 1, ..., n, where likelihood function is represented as a product of independent probability density functions:

$$L(c, \theta) = \prod_{i=1}^n f_{X(t_i) - X(t_{i-1})}(\delta_i) = \prod_{i=1}^n Ga(\delta_i | c[t_i^b - t_{i-1}^b], \theta) \tag{7}$$

$$= \prod_{i=1}^n \frac{1}{\Gamma(c[t_i^b - t_{i-1}^b]) \cdot \theta^{c[t_i^b - t_{i-1}^b] - 1}} \delta_i^{c[t_i^b - t_{i-1}^b] - 1} \exp\left(\frac{-\delta_i}{\theta}\right)$$

By taking the logarithm of the likelihood function, we obtain the following expression:

$$l(c, \theta) = \log L(c, \theta) = \sum_{i=1}^n [c[t_i^b - t_{i-1}^b] - 1] \log \delta_i - c \sum_{i=1}^n [t_i^b - t_{i-1}^b] \log \theta - \sum_{i=1}^n \log \Gamma(c[t_i^b - t_{i-1}^b]) - \frac{1}{\theta} \sum_{i=1}^n \delta_i \tag{8}$$

According to first partial derivatives of log-likelihood function of the increments with respect to c and θ, the maximum-likelihood estimates c and θ can be obtained based on the following system of equations:

$$\frac{\partial l(c, \theta)}{\partial c} = \sum_{i=1}^n [t_i^b - t_{i-1}^b] \{ \log \delta_i - \psi(c[t_i^b - t_{i-1}^b]) - \ln \theta \} = 0$$

$$\frac{\partial l(c, \theta)}{\partial \theta} = \frac{1}{\theta^2} \sum_{i=1}^n \delta_i - \frac{c}{\theta} \sum_{i=1}^n [t_i^b - t_{i-1}^b] = 0 \quad (9)$$

where: the function  $\psi(\cdot)$  - digamma function.

It is very common in practice that there is no sufficient available data on the condition of some structures. Therefore, we believe that is a very interesting comparative prediction of future deterioration based on the frequency of periodic inspections. We conducted parameter estimation using maximum likelihood method based on the available data from periodic inspections carried out every year, every second and every fifth year. In this instance we used inspection data of up to year 20 to estimate shape and scale parameters of the gamma process.

Tab. 2: Gamma process parameters for considered TCC beam estimated using different available data.

Type of inspection	Shape parameter	Scale parameter
	$c$	$\theta$
Every year	86.663	0.0242
Every second year	50.5524	0.0415
Every fifth year	39.7465	0.0528

## RESULTS AND DISCUSSION

All structures are designed to be safe and to satisfy certain predefined requirements through their service life. The accurate assessment and modelling of the structural lifecycle performance presents the continuous challenge for both design and maintenance engineers. Various deterministic deterioration models for long-term TCC beams behaviour assessment have been presented in the literature by different authors, Fragiaco and Cecotti (2004), Fragiaco (2006), Jorge et al. (2010) and Kanócz et al. (2013). These prediction models are based on laboratory tests, conducted on certain number of specimens and they are not including any variations and uncertainty in model variables. Therefore, deterministic prediction models have certain limitations in real conditions and these models provide point estimation of the future condition of the structure. Being that the long-term behaviour of TCC systems is an uncertain process, it is more appropriate to use probabilistic deterioration model in order to take into account the uncertainty in component material properties and influences of environmental conditions. In order to ensure normal use of structure during its service life, periodic inspections are carried out for monitoring its deterioration. Gamma process model is based on inspection condition data and the aim of this paper is to present the possibility of this approach in assessing the further deterioration and estimation of service life.

### Deterioration prediction

Based on the evaluated parameters that define the gamma process we can assess and predict future deterioration of timber-concrete composite beam. We can have  $X(t)$  denote the deterioration at time  $t$ ,  $t \geq 0$ , and have the probability density function of  $X(t)$ , in conformity with the definition of the gamma process, be given by

$$f_{X(t)}(x) = Ga(x|k(t), \theta) = \frac{1}{\Gamma(k(t)) \cdot \theta^{k(t)}} x^{k(t)-1} \exp\left\{\frac{-x}{\theta}\right\} \tag{10}$$

with mean and variance

$$E(X(t)) = k(t) \cdot \theta, \quad Var(X(t)) = k(t) \cdot \theta^2 \tag{11}$$

With the increasing number of inspections, we get more data on the state of the observed structures. Therefore, we will have a better estimation of gamma process parameters and the prediction of deterioration will be more precise, that is shown in Fig. 3.

Fig. 4 demonstrates the use of Gamma process representation to predict expected deterioration  $X(t)$  at different time according to the prediction from year 20.

Based on the analysis which results were presented at the previous figure, we could determine the probability that the observed composite beam will not reach serviceability limit state in certain years.

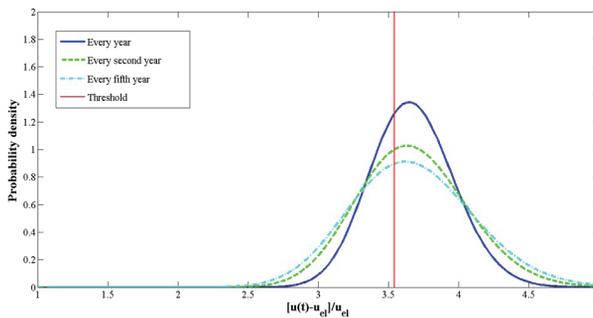


Fig. 3: Comparison of expected deterioration  $X(t)$  at year 60, using the prediction based on various available data.

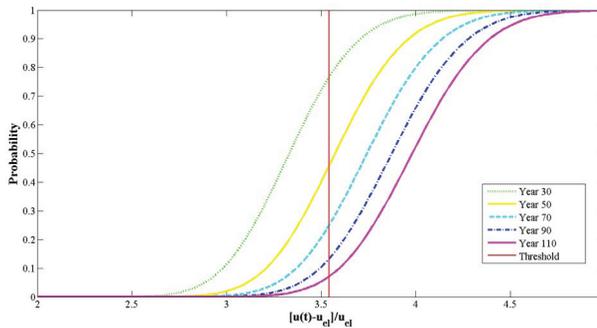


Fig. 4: Cumulative distribution function of deterioration  $X(t)$  in different years.

Tab. 3: Probability that considered beam will not reach serviceability limit state in certain years.

Lifetime	Year 30	Year 50	Year 70	Year 90	Year 110
Probability (%)	76	44	25	13	7

### Service life prediction

Maintenance management deals with the uncertainty in service life as the most important part of that process. Service life analysis is done depending on the maximum time at which the expected deterioration will exceed the service life threshold. Insurance companies as well as stake holders will all benefit from the estimate of the time when one structure reaches its serviceability limit state. We can define the service life  $T$  as the first time when the sample path of  $X(t)$  exceeds the predefined threshold  $\rho$ . Considering the deterioration as stochastic gamma process, the cumulative distribution function of service life can be presented as:

$$F_T(t) = \Pr(T \leq t | X(t) \geq \rho) = \int_{x=\rho}^{\infty} f_{X(t)}(x) dx = \frac{\Gamma(k(t), \rho / \theta)}{\Gamma(k(t))} \tag{12}$$

where:

$$\Gamma(a, x) = \int_{t=x}^{\infty} t^{a-1} e^{-t} dt \tag{13}$$

is the incomplete gamma function for  $x \geq 0$  and  $a > 0$ .

The survival function gives the probability that the considered structure will survive beyond specified time. This function is defined as:

$$S(t) = 1 - F(t) \tag{14}$$

Cumulative distribution function and survival function of service life for certain timber-composite beam according to previous estimated parameters for gamma process are shown in Fig. 5.

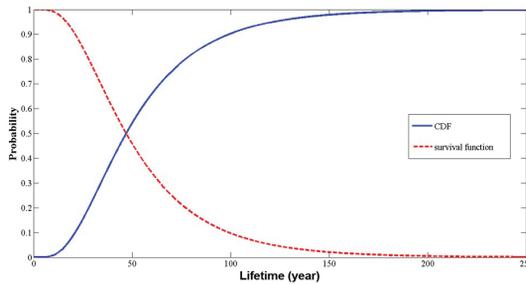


Fig. 5: Cumulative distribution function and survival function for service life.

In order to successfully plan a maintenance of the structure, it would be interesting to determine the time when the considered structure should reach the serviceability limit state with a certain probability (5, 50 and 95 %) (Fig. 6).

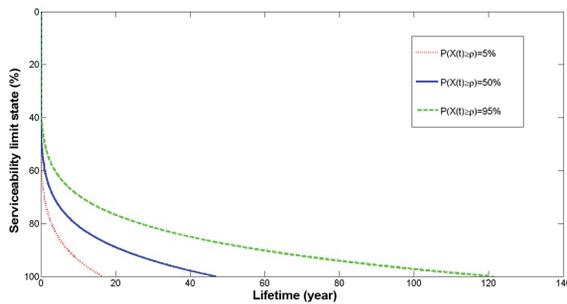


Fig. 6: Probability of achieving different levels of serviceability limit state condition during the lifetime.

Deterioration prediction models are commonly used in maintenance management of civil engineering structures. Presented approach can be used for different purposes. Beside accurate service life prediction of structures in service, this approach give us the opportunity to carry out various cost-benefit analysis, to make an optimal maintenance schedule or to evaluate the effect of conducted maintenance actions.

## CONCLUSIONS

Deterministic deterioration models are widely used in maintenance management of structures. However, they have certain limitations in real conditions, because these models provide point estimation of the future condition of structures. The aim of this paper is to present new stochastic process model that can capture uncertainty and temporal variability associated with the deterioration of timber-concrete composite beams under sustained load. Application of gamma process model on considered composite beam is included in this paper. The effect of frequency of periodic inspections on deterioration prediction is considered. Probability that considered beam will not reach serviceability limit state in certain years, likewise probability of achieving different levels of serviceability limit state condition during the service life are presented.

## REFERENCES

1. Abdel-Hameed, M., 1975: A gamma wear process. *IEEE Transaction on Reliability* 24(2): 152-3.
2. Aboura, K., Samali, B., Crews, K., Li, J., 2009: Stochastic processes for modeling bridge deterioration. *Futures in Mechanics of Structures and Materials* (eds. T. Aravinthan, W. Karunasena, H. Wang), CRC Press / Balkema, Taylor and Francis Group, London, UK. Pp 533-538.
3. Edirisinghe, R., Setunge, S., Zhang, G., 2013: Application of gamma process for building deterioration prediction. *Journal of Performance of Constructed Facilities* 27(6): 763-773.
4. Fragiaco, M., Ceccotti, A., 2004: A simplified approach for long-term evaluation of timber-concrete composite beams. In: *Proceedings of The 8<sup>th</sup> World Conference on Timber engineering 2*: 537-542, Lahti, Finland.
5. Fragiaco, M., 2006: Long-term behaviour of timber-concrete composite beams. II: Numerical analysis and simplified evaluation. *J. Struct. Eng.* 132(1): 23-33.

6. Frangopol, D.M., Kallen, M., van Noortwijk, J.M., 2004: Probabilistic models for life-cycle performance of deteriorating structures: Review and future directions. *Structural Engineering Materials* 6(4): 197-212.
7. Haowei, W., Tingxue, H., Qiaoli, M., 2015: Lifetime prediction based on gamma processes from accelerated degradation data. *Chinese Journal of Aeronautics* 28(1): 172-179.
8. Jorge, L.F., Schänzlin, J., Lopes, S.M.R., Cruz, H., Kuhlmann, U., 2010: Time-dependent behaviour of timber lightweight concrete composite floors. *Eng. Struct.* 32(12): 3966-3973.
9. Kanócz, J., Bajzecerová, V., Šteller, Š., 2013: Timber-concrete composite elements with various composite connections. Part 1: Screwed connection. *Wood Research* 58(4): 555-570.
10. Nicolai, R.P., Dekker, R., van Noortwijk, J.M., 2007: A comparison of models for measurable deterioration: An application to coatings on steel structures. *Reliability Engineering and System Safety* 92(12): 1635-1650.
11. Van Noortwijk, J.M., 2009: A survey of the application of gamma processes in maintenance. *Journal of Reliability Engineering and System Safety* 94(1): 2-21.
12. Velimirović, L., Perić, Z., Nikolić, J., Stanković, M., 2013: Design of compandor based on approximate the first-degree spline function. *Electronics and Electrical Engineering* 19(10): 151-154, ISSN: 1392-1215.
13. Yeoh, D., Fragiaco, M., Francheschi, M.D., Boon, K.H., 2011: The state-of-the-art on timber-concrete composite structures - A literature review. *J. Struct. Eng., ASCE*, 137(10): 1085-1095.

TATYANA MICIC  
CITY UNIVERSITY LONDON  
NORTHAMPTON SQUARE  
LONDON, EC1V 0HB  
UNITED KINGDOM

RAGOSLAV STOJIĆ  
UNIVERSITY OF NIŠ  
FACULTY OF CIVIL ENGINEERING AND ARCHITECTURE  
ALEKSANDRA MEDVEDEVA 14  
18000 NIŠ  
SERBIA

MIOMIR STANKOVIĆ  
UNIVERSITY OF NIŠ  
FACULTY OF OCCUPATIONAL SAFETY  
ČARNOJEVIĆA 10A  
18000 NIŠ  
SERBIA

NIKOLA VELIMIROVIĆ  
UNIVERSITY OF NIŠ  
FACULTY OF CIVIL ENGINEERING AND ARCHITECTURE  
ALEKSANDRA MEDVEDEVA 14  
FACULTY OF SCIENCE AND MATHEMATICS  
VIŠEGRADSKA 33  
18000 NIŠ  
SERBIA  
Corresponding author: [velimirovic.nikola@gmail.com](mailto:velimirovic.nikola@gmail.com)